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Investigation of the Transformation of Linear Function from the Perspective of Realistic Mathematics Education

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Abstract: In this study, it was aimed to investigate the processes of students' exploring a mathematical situation in daily life and examining and expressing this situation mathematically. The theoretical basis of the study is the realistic mathematics education framework. The research was designed as a case study and was carried out with the participation of pre-service elementary school mathematics teachers. In order to examine the horizontal-vertical transformation of linear function from the perspective of realistic mathematics education, an activity with two parts and three stages in each part was prepared. In the activity, the technologies of motion detector and graphing calculator were used to collect position-time data on walking motion and to obtain a function graph with these data. During the implementation of the activity, the students were observed and observation notes were taken. Semi-structured interviews were conducted with the participants. The data derived from the responses given by the students during the activity, the observation notes of the researcher and the interviews were subjected to content analysis. The results revealed that although the students did not have any problems while transforming an action in daily life into a function, it was observed that they had difficulty in carrying the transformation of this function into real life.

Keywords: *Realistic Mathematics Education, Linear Function, Transformation, Motion Detector, Graphing Calculators*

1. INTRODUCTION

Mathematics is a discipline that we encounter in every corner of our lives. When even the processes that we encounter in daily life and which do not seem to be related to any mathematical process at first glance, are examined in depth, it is seen that mathematical explanations can actually be made for these situations (Büyükikiz-Kütküt, 2017). In the examinations made on the relationship between daily life and mathematics, the proceeding starts with understanding the real situation intuitively then continuities with the finding patterns that can be represented by words or symbols. Afterwards, it is necessary to transfer the situation to the world of mathematics, transform it into a symbolic form and be able to make mathematical inferences (Pınar, 2019). In other words, in the mathematical analysis of real-life

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situations, the process of transition from informal knowledge to formal knowledge is observed. One of the mathematics education approaches that shed light on this process is Realistic Mathematics Education (RME) and this approach was introduced by Hans Freudenthal in 1971 (Alacacı, 2016).

The main idea of RME is seen to ensure that the concept or situation to be learned is shaped in the minds of students with the experiences in their own lives and to establish a connection between their experiences and the concept (Van Den Heuvel-Panhuizen, 2003). To do so, it is necessary to accept real life situations as the starting point. The purpose of the RME approach is to bring the information close to reality so that it can be visualized in the mind. This approach takes real life as the starting point. The fact that real life is taken as the starting point of RME is its most original aspect that distinguishes it from other teaching methods (Korkmaz, 2017).

Freudenthal, the founder of the RME approach, opposed anti-didactic approaches (Kütküt-Büyükikiz, 2017). He argued that it is not right to give the student a new knowledge that he/she has never encountered before and then ask him/her to apply it; instead, mathematics should start with real life situations, real life should be mathematized and thus formal knowledge should be reached (Altun, 2008). According to Freudenthal, mathematics is taught to be used in everyday life (Kaylak, 2014). Thus, mathematics is associated with reality and can be made close to human values. Educators should also develop a context for the concept to be taught in mathematics teaching and help students find a mathematical equivalent of the concept in their minds and shape that concept in their minds (Freudenthal, 1983). According to this approach, educators are expected to pose questions that will make students think in the lesson plans to be prepared by choosing appropriate daily life situations. Moreover, educators need to make students feel the strategies and create learning processes that will help them reach the mathematical concept starting from the daily life situation.

According to Freudenthal, the process of reaching a mathematical concept starting from daily life situations is called “mathematization”. The mathematization process consists of two separate components defined as horizontal and vertical mathematization. Horizontal mathematization is transforming a problem in daily life into a mathematical problem, diagnosing, defining, schematizing and formulating original mathematics with personal methods different from daily life situations (Korkmaz, 2017). Vertical mathematization is the self-regulation of the mathematical process (Treffers, 1988). Expressing mathematical knowledge in mathematical terms is restating the relationship within a formula, organizing, formulating and generalizing a mathematical model. Handling vertical mathematization first and then horizontal mathematization is called progressive mathematization. The purpose of RME is to improve the student’s knowledge of mathematics through progressive mathematization (Alacacı, 2016).

As seen in Figure 1 below, horizontal mathematization in RME represents the relationship established between the mathematical model and the real context in daily life situations. In addition, in Figure 1, vertical mathematization is expressed as the processes of interpreting the mathematical model within the framework of mathematical relations (Gravemeijer, 1994).

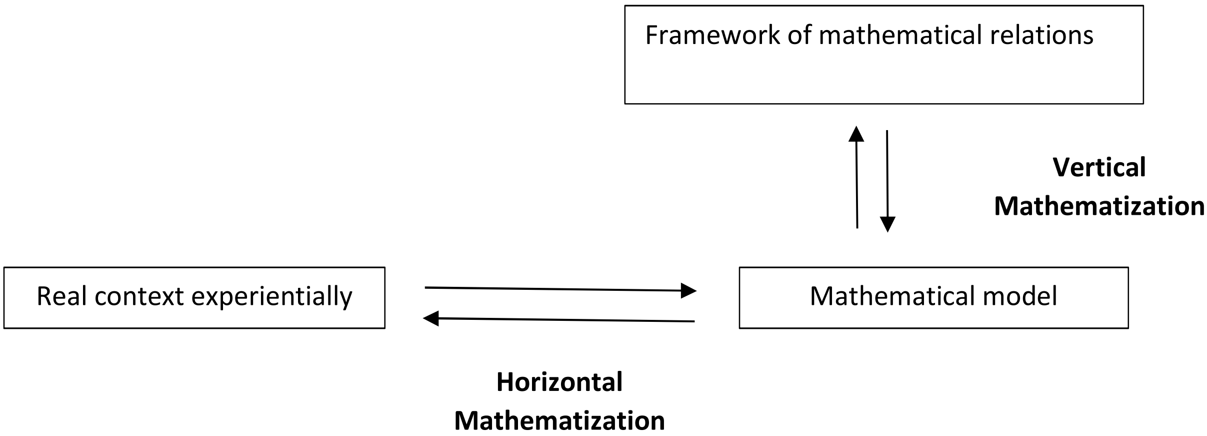

Figure 1. *Horizontal and Vertical Mathematization (Gravemeijer, 1994)*

There are some technological tools that will make it easier for students to analyze the elements of daily life mathematically and transfer them to the symbolic language of mathematics (Flores, 2002). Detectors, graphing calculators and pocket computers are examples of these technological tools. Through technological tools, students can record the data they have obtained from daily life situations and make mathematical analyses based on these data (Erbaş, 2005; Güveli & Baki, 2000). The mathematical concept that is frequently encountered in these analyses is the concept of function. In other words, this concept, which is used to observe the change of two variables depending on each other and to make mathematical inferences, is a basic and unifying concept of mathematics (Leinhardt, Zaslavsky, & Stein, 1990).

There are many elements that support learners to make sense of the concept within the subject of function such as the definition of the function, the properties of its algebraic representation and drawing its graphics. From among them, transformation as an important operation of the function, that is, the translation and rotation of the graph of the function in axes, is a necessary operation in order to better understand and interpret the function. The importance of function transformation is emphasized in mathematics education standards and curricula in many countries (Common Core State Standards for Mathematics (CCSM), 2010; National Council of Teachers of Mathematics (NCTM), 2000; Ministry of National Education (MoNE), 2018). In NCTM standards, students are expected to use different representations to understand function transformation (NCTM, 2000). A great care is taken for students learning transformation to perceive the resulting transformation on the basis of motion and to see that it has created a new function (Flanagan, 2001). In our country, there are procedures and objectives in the context of transformation at the secondary and high school level in the mathematics curriculum. For example, at the 8^{th} grade level, regardless of the subject of function, there are objectives related to geometric shapes and the transformations of the points, which are the building blocks of these shapes. At the secondary education level, besides the geometry objectives, there are also objectives about the transformation of functions at the 10^{th} grade level. Within the context of the linear functions and the analysis of these functions, the concept of transformation is included in the 10^{th} grade objectives at the secondary education level. At the same time, it is aimed to transform the quadratic function in 11^{th} grade learning objectives and to obtain new algebraic and graphical functions.

Function transformation helps to learn about the concept of function, as well as providing the opportunity to reflect the function and use it in different situations (Lage & Trigueras, 2006). In order for the transformation to take place, it is necessary to take the composition of the functions and transform them into a new function. Addition, subtraction, multiplication, division and composition operations applied to a function enable the graph of the function to progress in the horizontal or vertical direction and transform it into another function. In this context, the transformation of the function allows obtaining more than one new function from a function by adhering to the main function. This can create the opportunity for conceptual richness and learning new concepts in connection with each other.

The type of function that students first encounter in their education life and through which they work on the concept of transformation is the linear function. Linear functions enable students to examine the relationship between two variables and to gain first experience that will contribute to their mathematical development (Pierce, 2005). Linear functions are algebraic representations that allow the study of variables with a constant rate of change.

Students generally perceive the operations performed during function transformation as a new formula that emerges by associating with the action of transport in the coordinate system (Graham, Ferini & Mundy, 1990; Breidenbach, Dubinsky, Hawks, & Nichols, 1992). It is observed that the main reason for this perception is that in most of the problems related to functions in mathematics lessons, students consider the function as a formula and do not focus on changes (Even, 1993; Vinner & Dreyfus, 1989). It is stated that the preferred teaching method has an effect on the student's learning of the concept of function (Ural, 2006). It has been seen that some methods that can be preferred in the teaching of function transformation can motivate students to memorize the transformation rules rather than accomplish conceptual learning on this subject (Hall & Giacin, 2013). However, the student needs to think deeply about function transformation and make connections. The use of technology is also recommended in the teaching process so that students can make connections between the functions that emerge as a result of the transformation (Boz & Erbilgin, 2015). It is emphasized that technology can also have some aspects that foster mathematical thinking and encourage cooperation in the learning process (Roschelle, Pea, Hoadley, Gordin, & Means, 2000). Students' performing mathematical tasks in cooperation allows them to explore different ideas and to work on their mistakes by noticing them (Johnson & Johnson, 2008).

According to Tall (1997), functions are a useful mathematical concept used to show how events change. Therefore, it is emphasized that it will be more meaningful to examine the changes observed in daily life events with mathematical models in the process of teaching functions (Ural, 2006). In this context, a study including mathematical situations that could be visualized in the mind and revealing the process of mathematization on the basis of the RME approach seems to be highly useful. Therefore, in this study, the way of reaching mathematical knowledge was designed based on the real model, such as act of walking. In the literature review, it was seen that there are not enough studies that require the examination of function transformation in relation to the act of walking which is a real life situation in the context of the RME approach. Thus, this study contributes to understanding on function transformation literature through real life situation. Since the understanding of function transformation is one of the important concept for learning function, this study shines a light on transformation of linear function through the lens of RME.

1.1. *Purpose of the Study*

In this study, it was aimed to determine the problems experienced by third-year university students in expressing the act of walking in daily life as a function. Transformation processes were also used in the function determination stages. In this regard, the following questions guided the study:

1. What is the pre-service middle school mathematics teachers' prior knowledge about a linear function and the translation of this function as a type of transformation?
2. What is the state of the pre-service middle school mathematics teachers in terms of transforming a linear function on x and y axes and obtaining the formula of new functions by conducting individual work?
3. What is the state of the pre-service middle school mathematics teachers in terms of transforming a linear function on x and y axes and obtaining the formula of new functions by conducting group work?
4. What is the state of the pre-service middle school mathematics teachers in terms of converting the act of walking at constant speed into a linear function?
5. What is the state of the pre-service middle school mathematics teachers in terms of explaining the transformed function as the act of walking?

2. METHODOLOGY

2.1. Research Model

In this study, the case study design, one of the qualitative research methods, was used. Case studies provide the opportunity to examine a program, event, activity, process or one or more individuals in depth (Creswell, 2009). In this study, the ability of a group of students to perform function transformation with the help of technology, the problems they experienced while doing this and their working environments were examined. Since the aim of the study is to examine understanding of the students' perspective on the function transformation the holistic single case design was used. The whole students and the activity applied to them are considered as one case, not each student is taken as a case. Therefore, instead of comparison among the cases; describing and presenting the picture as a whole is preferred. Moreover, the processes involved in the students' performing transformation were taken as the analysis unit.

2.2. Study Group

The study was carried out with the participation of 4 students studying at the department of middle school mathematics teaching of an education faculty and the students participated in the study on a volunteer basis. A pilot study was conducted on one male student who was studying in the same department and was not involved in the main study. The participants were determined by using the convenience sampling method so that the participants could be easily available to the researchers. In the study, the participants were asked to evaluate their technological propensity on a scale ranging from 1 (little) to 5 (too much) due to the use of technological tools in the study. What is meant by the propensity towards using technology here is the willingness of the students to use any digital application or different technology that they encounter for the first time. It also means exhibiting an enthusiastic and positive attitude towards the use of technology in mathematics teaching and learning. In addition, when the grade point averages of the students participating in the activity were examined, it was observed that they were very close to each other (between 3.20-3.36 out of 4), so it was not necessary to examine their academic achievements in the analyses to be made. In the table below, the gender of the students and the degree of their technological propensity are given.

Table 1. *Demographic Information of the Participants*

	Gender	Technological propensity (1-5)
Student-1 (S1)	Female	4
Student-2 (S2)	Female	2
Student-3 (S3)	Female	2
Student-4 (S4)	Male	4

2.3. Data Collection Tools

The activity sheet prepared by the researchers is the main data collection tool for the study. The formation of this activity, context of it and how the activity conducted with students are explained in the next section.

Besides the activity sheet, the second data collection tool is the observation notes of the researchers. The two researchers closely followed the students' work processes and recorded the students' work by taking written notes about their observations and video recording. During the activity application session, students were discussing in the group and the researchers joined the groups to listen students' ideas without any contribution. They took short notes about how students use the movement detector and graphing calculator to draw the expected function, how they conceptualize transforming the function and how they interpret the transformed function into the real life walking situation. While students studying on the activity sheet a camera recording the classwork.

Additionally, after five months from the application, the first researcher interviewed with four students about function transformation. Their activity sheet was given them and asked that whether they remembered the activity. They were asked about the mathematical ideas of the activity and questions on function transformation. These interviews were not recorded but yet the researcher wrote down the summary of the interview.

2.4. Process of the Formation of the Activity

An activity was prepared to be implemented in the research process (App. 1). The data of the study were derived from the responses given by the students during the activity. The responses given by the students to the questions during the activity were examined step by step and on the basis of the explanations made by the students, how the daily life situation was used in the teaching delivered and how the cooperation between the students was supported with technology were examined.

The activity was developed by the researchers. The aim of the activity whose piloting was carried out was to enable the participants to express the act of walking mathematically using graphing calculators and motion detectors. It also examined the students' mathematization processes. The act of walking addressed in the current study would be transformed into a linear function and the questions in the textbook of Kamischke, Murdock, and Kamischke (2002) (Movin' Around) were used in the development process of the activity, which was designed on examining the horizontal (in the x-axis direction) and vertical (in the y-axis) transformations of this function. The textbook includes questions about graphing calculators and function creation processes. In the current study, the questions in the textbook were improved and deepened. As a result of the pilot study, the questions in the activity were divided into two parts as vertical and horizontal transformation and new additions were made to the examinations carried out through the motion detector.

The activity consisted of two parts in the pilot study. These are the vertical and horizontal transformation stages. In the first of these stages, there are questions to check the prior knowledge and in the second, there are questions about the data to be obtained using a graphing calculator and motion detector. The observation notes obtained at the end of the pilot study and the opinions of the field expert were examined. On the basis of these opinions, the activity was organized in 2 sections and 3 stages in each section in a way to examine horizontal and vertical transformation separately. As a result, the following stages are included in the final version of the activity. The activity sheet is designed to reflect the main idea of RME such that mathematics as a human activity. In this idea mathematizing is the key concept. Therefore, in the prepared activity walking action is observed through movement detector and try to understand the data with graphing calculator. The questions on the activity sheet help students in their mathematizing processes. The entire activity is in Appendix 1:

Section 1: Examination of Vertical Transformation

1st Stage: The knowledge of the students about linear function and vertical transformation was questioned. The processes of plotting the graph of a given line equation and its transformation on the y-axis were examined. Pen and paper were used in these processes.

2nd Stage: The students' processes of discovering the linear function given to them from different walks using a graphing calculator and motion detector were examined. In the first stage, they were expected to perform walking motions to attain the functions they created by making vertical transformations. In this stage, graphing calculators and motion detector were used.

3rd Stage: As shown in Figure 2 below, two students were allowed to examine using motion detectors at the same time and walking instructions were given for them to observe vertical transformations. They were expected to write the resulting functions as a result of these instructions.

Figure 2. *Photo Showing the Data Collection Process (Kamischke, Murdock, & Kamischke, 2002, p. 187)*

Section 2: Examination of Horizontal Transformation

All the steps in section 1 were repeated, this time focusing on translations along the x-axis.

In both sections, the first stage was designed to understand the students' prior knowledge of the concept of transformation and the use of technology and daily life situations were examined in the second and third stages. The activity lasted for 4 class hours. While the students were expected to work individually in the first two stages of section 1 and section 2, all of the participants worked together in the third stage. Two months after the implementation of the activity, the information of the participants about the transformation was questioned again (Appendix 2), and the retention of the activity was evaluated.

2.5. Data Collection Process

At the beginning of the application, the activity was distributed to the students. In the initial stages, the material was used to check the line equations drawn using only prior information. These stages are the stages in which the students were expected to reveal their prior knowledge and they were called the "*prior knowledge control stages*".

In the second stage, the materials were distributed to the students and the students were expected to answer the questions in this stage using the materials. For the students who encountered the materials for the first time, 15 minutes was given for them to examine the materials at the beginning of the second stage. Information was given about the tabs and keys that the students would use. Students let observe some movement by using motion detector and draw graphs of these movements on graphical calculator to identify the changes of the graphs. In this stage, the students also used the advantages provided by the materials. The materials provided the opportunity to construct functions and identify their new states having undergone transformation. Therefore, this stage was called the "*Stage of Individual Discovery*".

In the third stage, the students were expected to walk according to the instructions given in cooperation. With the data obtained as a result of the walk, they were expected to perform the tasks of drawing graphs, filling in tables and generalizing found in the 3rd stage. Therefore, this stage was called the "*Stage of Discovery with a Group*".

After 2 months, a second set of interviews were conducted with the two students to check the retention of this activity. Two-stage questions (Appendix 2) addressing horizontal and vertical transformation were prepared to be used in the interviews. In these interviews, a function equation was given to the students based on their work in the activity. The students were expected to discover the motion that could be the equivalent of this function equation in daily life. Moreover, questions were asked to the students about the application of the transformation process on the given function and about the determination of the new motion that would emerge after the transformation. Therefore, this stage was called the "*Stage of Persistence in Mental Associations*". The sections in the activity and the stages within the sections are given in Table 2 below.

Table 2. *Information about the Sections and Stages of the Activity*

	Stages	Content
Vertical/Vertical Transformation	Stage of Checking the Prior Knowledge	The process of answering the function questions directed using pen-paper
	Stage of Individual Discovery	The processes of recognizing technological tools and discovering the function within walking with the help of tools
	Stage of Discovery with a Group	The process of performing the walking instructions and examining the obtained functions
	Persistence in Mental Associations	The process of examining different functions in order to examine the persistence in mental associations

The stages of the activity are evolved according to RME principles. First of all, students' prior knowledge on transformation is checked so that the mathematical investigation in the activity make sense for the students. After that the individual examination is applied in case students can make horizontal mathematizing process. The stage of discovery in the group is conducted for vertical mathematizing can be produced by the students in a group working procedure. At last, the persistence in mental association stage is applied to understand whether students still remember what they did two months ago and the idea of transformation is meaningful for them.

2.6. Data Analysis

The study has 3 main data sets. These are the participants' worksheets, the observation notes of the first researcher and the transcripts of the interviews made for the purpose of examining retention. The worksheets were analyzed using a descriptive analysis framework based on function transformation. Furthermore, observation notes were used to support the data obtained from the activity. The activity sheets and the observation is analyzed through the first three stages of the framework which is prepared based on the stages of the activity (see table 2). Additionally, the interview data is analyzed through the last stage of the framework. In the framework there are four themes and related codes given below.

Table 3. *Themes and Codes List*

Stages of the Activity	Themes	Codes
Stage of Checking the Prior Knowledge	Knowledge about drawing Linear Function	Intersection points Slope
	Knowledge on transformation	New graph of transformed function New rule of transformed function
Stage of Individual Discovery	Understanding the motion detector working principle	Creating a linear function by walking with motion detector
	Vertical transformation of linear function	Reproduce vertically shifted graph by walking
	Horizontal transformation of linear function	Reproducing horizontally shifted graph by walking
Stage of Discovery with a Group	Producing the formula of the transformed functions	Using gathered data to produce formula of transformed functions
Persistence in Mental Associations	Persistency of knowledge on transformation	Remember the relation between walking and function transformation

The reliability of the study is considered in four perspectives. The first one is inter-coder reliability. The analyses of the data obtained from the participants' worksheets were made separately by both researchers. The researchers who came together reached a consensus by discussing the findings they obtained. The second perspective for the reliability is respondent validation. The findings are shared with the respondents who are participating in the study for obtaining the credibility of the results. The third perspective of reliability is presenting direct quotation of the participants from their answers either from activity sheet or from the interview. Lastly, the data analysis and reporting the results are constructed on a theoretical framework which is realistic mathematics education.

3. FINDINGS

In the current study, the act of walking that we encounter in daily life was examined with technological tools. By examining the act of walking as a function, the students' perspectives of the concept of transformation of functions, their way of making sense and their perspectives of mathematical analysis were evaluated. In this connection, the findings were examined under 4 sub-headings: The Stage of Checking Prior Knowledge, The Stage of Individual Discovery, The Stage of Discovery with a Group and The Persistence in Mental Associations.

3.1. The Stage of Checking Prior Knowledge

It was observed that answers based on rote learning were generally given to the questions asked to evaluate the students' prior knowledge about the drawing of functions and the transformation of a function on axes.

Although they were able to easily draw the function graphs given to them at first, it was noticed that they did not perform the conceptual interpretations and operations related to the transformation when it was said, "Translate the graph you obtained 3 units up, 2 units to the right and find the equation by drawing the new graph". It was seen that they made an effort to remember the information they saw and memorized in their past learning instead of conceptual interpretations. It is possible to see this situation in the following conversation between two students.

> *S1: It has a rule, what is it? Are we adding the number to x or y?*
>
> *S2: While translating up, we need to subtract from y. The subtracted value is carried to the other side of the equation as +. Therefore, y=2x+3.*
>
> *Researcher: "While drawing the line, did you draw it with this equation in mind?*
>
> *S1: No, I moved it on the axis by the specified unit without distorting the line.*

In Figure 3 below, the drawing produced by S3 for a given equation and the drawings he/she obtained for the graphs after the transformation of this function are given. It is noticed that he/she determined points in the drawing of the graph and drew the line passing through these points.

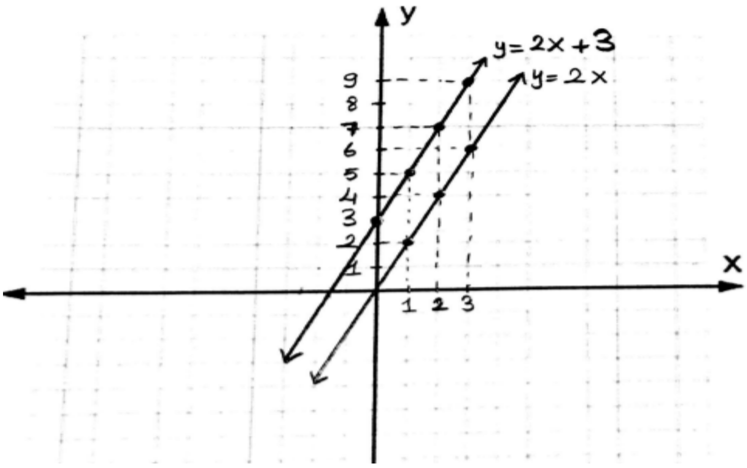

Figure 3. *The Response Given by S3*

During the question asked for them to transform the graph whose equation was given, the students were observed to be trying to remember and discuss the rules of transformation by talking among themselves. When the students were expected to explain the equation they obtained after the transformation process, it was noticed that the students made explanations on the graph without mentioning these rules. While explaining the method they used in the process of creating the equation, the students gave answers that included connections and relations rather than information foreign to them. This coincides with the main idea of RME. The students were observed to pay attention to selecting a point on the line that cuts the coordinate system and moving it by the given unit on the axis and having the same parallelism and slope between the moved line and the new line. It was seen that S3 only made changes in the equation according to the transformation rules she had memorized during the vertical transformation stage and checked the equation he/she obtained by looking at the parallelism and slope of the lines. It was noticed that the same student did not prefer the same method in the horizontal transformation section and answered the question by using unit squares.

> *Question: What is the equation of the new line? How did you find this line equation? Please explain.*
>
> *Answer: y=2x+3 I determined a point where the line cuts y and x points. I used unit squares. Moreover, since it shifted 3 units upward, I added 3 to the line.*

Additionally, it is seen that a student made changes on the equation with the information he/she remembered during the vertical transformation stage and controlled the transformation process by taking into account the slope and parallelism. This can be observed from the answer of the S3 for the question given below:

Question: What is the equation of the new line? How did you find this line equation? Please explain.

Answer: y=2x+3 I obtained the transformed form by making changes on the equation of the line. As they are parallel, their slopes remain the same. I took this into consideration.

For the same question, S3 first transformed during the horizontal transformation stage, handled the x and y values and performed the transformation by taking into account the slope and parallelism.

Question: What is the equation of the new line? How did you find this line equation? Please explain.

Answer: y=2x-4 As these lines are parallel to each other, the slope of the new line is the same. Then, I wrote an equation according to the y values corresponding to the x values.

In other words, the students' prior knowledge did not emerge depending on conceptual interpretations. The students used their memorized knowledge, but also tried to explain the transformation by making associations and basing it on the concepts of slope and parallelism.

3.2. The Stage of Individual Discovery

The stage of individual discovery consists of 3 stages. In the stage of individual discovery, first, it is examined how the act of walking performed by the students will create a graph with the motion detectors. Then, the situations that occur as a result of performing re-walking in order to transform the obtained function on both vertical and horizontal axis are examined. The findings regarding these states are presented under 3 sub-headings:

3.2.1. Use of the Motion Detector in Graph Drawing in the Stage of Individual Discovery

While only graphing calculators were used in the stage of checking prior knowledge, motion detectors were also used together with graphing calculators in the second stage. In the stage of checking prior knowledge, the students were expected to think about what linear functions correspond to in daily life. In other words, they were asked to form the equations given with the act of walking. Therefore, like horizontal mathematization in RME, the students were expected to think about how a situation in daily life can be expressed mathematically. In the questions posed in this direction, the students were expected to try different walking styles, examine the graphic representations of the data they obtained from these walks and discover the desired graph by making walking trials. In addition, they were expected to use position-time data and to realize that walking must occur with the act of constant-speed walking in order to form a linear graph.

In this stage, it was aimed to form the linear graph they drew for the equation given in the stage of checking prior knowledge with the act of walking. It was noticed that some students focused on the image of the graph they drew. It was observed that the students moved the motion detector diagonally from bottom to top, considering the graph they drew in the first stage. The movement made was the same as drawing the image of the graph in the coordinate system. After this movement, the desired graphics could not be obtained. It was observed that some students knew the linear graph as a figure, but they could not make a mental association about how to obtain this graph with the act of walking. Another observed behaviour was an effort to create a graph according to the position-time relationship by moving the motion detector forward-backward. This effort did not yield certain results for every student. The reason for this is the existence of another condition that must be taken into account. This condition was that walking should have a constant speed. Some students realized after a few trials that they needed to perform a constant speed walk. Some students, on the other hand, argued that the speed of walk should be increased as they initially regarded the graph as an ascending graph and thus they felt that they had to form an ascending graph. In this stage, as a result of the questions directed to the students such as "What kind of situations did you encounter during your experiment?" and "Can you discuss the situations that turned out to be wrong?", they were observed to perform a several trials. Some students were observed to realize that the graph is a linear graph through the effort they made to create the descending graph. This conceptual confusion emerging was resolved considering the graphics obtained through the students' trials and observations. Although walking of an equation given in the question was asked for, the preferred exploration method was to try various walks in daily life and to examine the graphs that emerged from these walks and then the exploration of the equations by the students. This preferred method can be addressed under horizontal mathematization in RME.

When the question about using motion detector one of the students gave an incorrect explanation as follow:

> *Question: What kind of walk do you need to perform in order to create the y=2x line given in the 1st question of Part A by using the motion detector?*
>
> *Answer: After I fixed the motion detector, I rapidly moved it away. As the graph was an ascending graph, I performed acceleration.*

For the same question, it is seen that a student (S2) came to the conclusion that he/she should perform the act of constant speed walking as a result of his/her discoveries as saying: "*I fix the motion detector somewhere. Then I move it steadily at a constant speed from the position I fix it*." In other words, in this stage, slightly different from the RME approach, the students were expected to make associations between their prior knowledge and real life. It was not started with a daily life situation, but a situation in daily life was mathematized and compared with his/her own trials. The students discovered the equation in daily life and decided how they should walk.

3.2.2. Vertical Transformation in the Stage of Individual Discovery

In the continuation of the individual discovery stage in the vertical transformation section, the students were expected to create the graph of the line they drew by transforming with the act of walking at a constant speed. In this stage, it was observed that some students used the position data included in the previous question and they did not have as much difficulty as they experienced in the activity of discovering the act of walking at a constant speed. The students discovered how to walk for transformation in a shorter time. It was seen in the dialogues between the student and the researcher that they took into account the information they obtained from the first question of the individual discovery stage and the graphics they drew during the stage of checking prior knowledge.

> *Researcher: What are the differences with the previous walk? How did you create this graph in the graphing calculator?*
>
> *S2: I changed my positions. I looked at the graphs I had drawn. There is a difference of 3 units between the first graph I drew in the 0^{th} second and the transformed graph. That is, the transformed graph is further away from the position in the 0^{th} second. I adjust the distance and walk again at a steady pace.*

In the dialogues, it was determined that the students tried to give answers to the problem by developing personal methods, as in the horizontal mathematization stage of RME, by looking at student responses and observation notes. The personal methods developed here are the discoveries on walk that start with their prior knowledge and trial and error, showing that they can create a linear line with the graphics they drew and with the act of constant speed walking. Using these discoveries, they themselves found the transformation operation of the graph, which they obtained with the given equation, in the act of walking. In the previous question, the students discovered which walks created which graphs by trial and error. In this stage, they associated their discoveries with the graphs they drew as a result of the transformation and were able to use their knowledge of transformation. Below there is a correct answer of a student interviewed by the researcher about the walk he/she took to create the new line he/she found by transforming using the motion detector.

> *Question: What kind of walk do you need to perform for the formation of the new line you found by transforming in the 2^{nd} question in Part A by using the motion detector? Try this way of walking. Explain the results.*
>
> *Answer: Firstly, the drawing of the graph appears upward when I increase the distance between motion detector and the point that I fixed. I mean the shape of the graph appears above the (0,0).*

It was observed that some students could not perform the walking of the graph obtained by transforming. Afterwards, it was determined that these students could not reach the correct answer in the first question in the stage of individual study. Thus, it can be said that each problem in the stage of individual discovery is related to each other. In addition, it can be interpreted from the responses of the student that he/she formed his/her ideas conceptually by associating them with daily life in the subjects of linear functions and transformation in a gradual manner. According to RME, the student should establish a connection between the concepts or situations to be learned and the experiences in his/her own life. The deficiency between the connections causes the relationship to be established regarding the concept not to be established. It was observed that the student, who could not reach the correct answer, looked at the graph drawn again while creating the graph of the transformed line on the graphing calculator, but could not fully relate to the position-time graph and thought that he/she should move it by bringing it to a certain height on his device because he/she translated it upwards. The explanation of the student experiencing this situation is given below:

> *Question: What kind of walk do you need to perform for the formation of the new line you found by transforming in the 2nd question in Part A by using the motion detector? Try this way of walking. Explain the results.*
>
> *Answer: For the first graph, I performed transformation on the ground. I thought that we need to move the motion detector from a certain height for this graph.*

In other words, in this stage, the students examined the changes in walk, position and time in order to carry out the vertical transformation within the walk. In the beginning, they realized that they had to change their distance from the detector and they performed their walk according to this situation. In this stage, some students' misconceptions were noticed because they focused on the shape of the graph.

3.2.3. Horizontal Transformation in the Stage of Individual Discovery

In the stage of individual discovery in the horizontal transformation section, the students were expected to create the graph they drew with the given equation on the graphing calculator by walking at a constant speed. Then, the students were asked to notice the relationship between the graph of the given equation and the graph they transformed and to explore the transformed graph in the act of walking. The students were asked the question "What kind of walk do you need to perform for the formation of the new line you found by transforming in the 2nd question in Part A by using the motion detector. Try this way of walking. Explain with its reasons".

In relation to this question, the answers given by the students and observation notes in the horizontal transformation section were examined. As a result of the examinations, it was concluded that the students tried to create these graphs on the devices by paying attention to the relationship between the graphics they drew in the first stage. Furthermore, it was observed that the students performed walking by taking into account the operations they did in vertical transformation. The students' associating a new situation with different fields can be explained with the principle of embeddedness in RME. In addition, according to the observation notes and student confirmations, it was noticed that the students had more difficulty in this discovery than they did in the vertical transformation section and they discovered it after a few trials. As an example of this situation, an excerpt from the interview with a student is as follows:

> *Researcher: How did you create the transformed graphs on devices?*
>
> *S4: First I thought I should adjust the position as in vertical transformation but this was a little different, it didn't happen right away. When we look at the region above the x-axis of the two graphs, the given graph passes through the point (0,0) and the transformed graph passes through the point (2,0). It looks as if it stopped for 2 seconds and then started moving. Considering this, I kept the motion detector fixed for a while and then moved it.*

In the following answer is given by the student with whom the dialogue took place.

> *Question: What kind of walk do you need to perform for the formation of the new line you found by transforming in the 2 nd question in Part A by using the motion detector? Try this way of walking. Explain with its reasons.*
>
> *Answer: First, I wait as long as the time equal to the unit I will translate and then I start the motion.*

Moreover, it was observed that a student who used devices in this process and was closely related to technology answered the question with a different perspective. The student said that a different method could be used, that he/she could create two graphs with two devices and compare these graphs and gave the answer as:

> *The measurement starts with two devices. The first device and the motion start simultaneously. The motion does not start in the second device but the device is started. It starts from the same position. Their seconds of starting the motion are not equal.*

S3 said that "*I do not know what kind of walking should be performed*" where it can be deduced that he/she could not comprehend the walking motion and responded according to his/her own inferences in the vertical transformation. In other words, in this stage, the students examined the changes in walk, position and time in order to carry out the horizontal transformation within walking. By using one or two devices, they realized that they had to wait for a while for the horizontally transformed walking and they performed their walking. Some students stated that they did not know what kind of walk to perform.

3.3. Stage of Discovery with a Group

In this stage, different walking instructions that the students could perform with a group in vertical and horizontal transformation were given. The reactions of the students to the situations that emerged as a result of the execution of these instructions were evaluated. Although the students evaluated the walking instructions by doing group work, they filled the tables individually using the data they obtained and created the graphs according to these tables. The students made inferences with the information they obtained as a result of their works. In addition, the students answered the questions at the end of the stage. This stage progressed faster than the stage of individual discovery. It was observed that a student who could not find answers to the questions in the stage of individual discovery answered the questions in this stage.

3.3.1. Vertical Transformation in the Stage of Discovery with a Group

All the members of the 4-person group took some responsibilities. Tables were filled and graphs were drawn with the data obtained by the students during their walk. In this stage of the activity students tried to observe motion detector and obtain data. In their worksheet the question “Examine the data collected in the calculators. Fill in the table below with the data you obtained from the detector.” is asked. One of the group filled out the table like below:

Table 4. *The Data Table Prepared by S2 for Vertical Transformation*

Time	The position of detector A	The position of detector A
T_1 second	0,100	0,900
T_2 second	0,300	1,100
T_3 second	0,500	1,300
T_4 second	0,700	1,500
T_5 second	0,900	1,700

In the same group students obtained from the graphing calculator depending on the data gathered by the detector. In the figure 4 one of the example of drawn graph by using data is given.

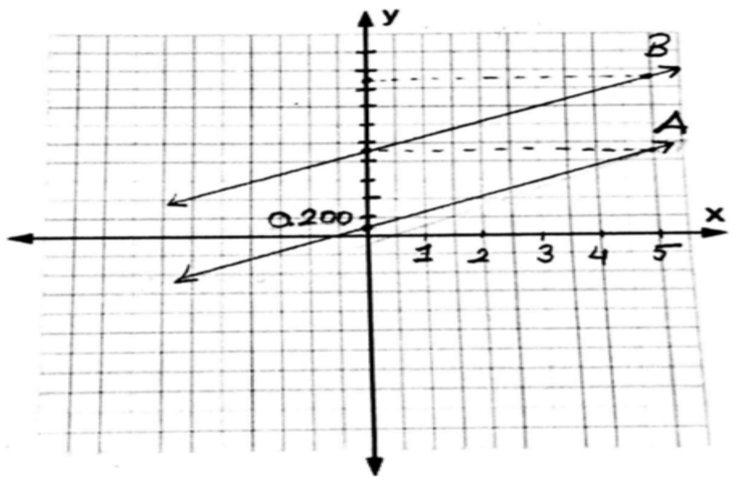

Figure 4. *The Answer Given by S2*

The students were expected to explain the graphs they created on the graphing calculator according to the instructions given. In this explanation, they were expected to use the concept of transformation and the following question was directed to the students "Using the two graphs and table data, explain the relationship between the graphs through the concept of translation". In this question, it was seen that the students explained the act of walking to be performed with instructions that required the use of two devices by associating the graphics they discovered during the stage of individual discovery and the concept of transformation using a single device. In addition, it was seen that the students explained using their inferences about the walk according to the cut points and position of the graph in the stage of individual discovery. However, in this stage, it was determined that the students were able to explain the real life situation of walking with a mathematical language, namely graphics. This preferred method is a horizontal mathematization according to Freudenthal's RME approach. In the following question the students were expected to explain the relationships between the walking data and the created graphs with the concept of transformation. No guidance was given to the students or no intervention was made to them. It is seen that the student (S2) preferred to explain the relationship between the graphs within walking in the question:

> *Question: By using the two graphs formed and the table data and the concept of transformation as well, explain the relationship of the graphs with each other.*
>
> *Answer: As the moving object has a constant speed according to the table data, the graph is linear; moreover, as the object had already proceeded when the detector B started to detect the motion, the graph formed is above the first graph.*

In this stage, it was expected that the relationship between the two walks would be abstracted and explained in mathematical language and expressed with equations. For this purpose, the question "*Let the graph created by person A be the y=f(x) function. In this case, how can the graph created by Person B be expressed in general?*" was directed to the students. S1 answered as "*If y=f(x) is the graph of Person A, then y=f(x) + 800 gives the function of Person B* *according to the data."* In the stage of individual discovery, it was observed that the students tried to establish a relationship between walking motions by looking at the graphs. In the stage of discovery with a group, they tried to answer this question by taking into account the tables they filled in by walking together. These tables include positions of the walks they performed. Some of the students concerned the relationship between functions is formulated according to their answer taking into account the distance between the walks. The students' mathematically expressing a daily life situation corresponds to vertical mathematization in RME. In other words, the acts of walking regarding vertical transformation were carried out with the instructions given in this stage. The students' working with a given walk and the effect of cooperation on their work were examined. The tables and graphics obtained after the walk were filled in and drawn individually and the relationships between the graphics were explained with the concept of translation.

3.3.2. Horizontal Transformation in the Stage of Discovery with a Group

In the third stage of the horizontal transformation, the tables were filled in and graphs were drawn with the data obtained during the walk carried out with each member performing some tasks in the group of 4 people. Differences were observed in the answers given by the students during the stage of discovery with a group in the horizontal transformation section. It was noticed that two of the students did not comply with the data of walking performed in cooperation. It was seen that two of these students predicted the graph and drew it with blunt lines in the coordinate system. The graph drawn by a student is given in Figure 5 below.

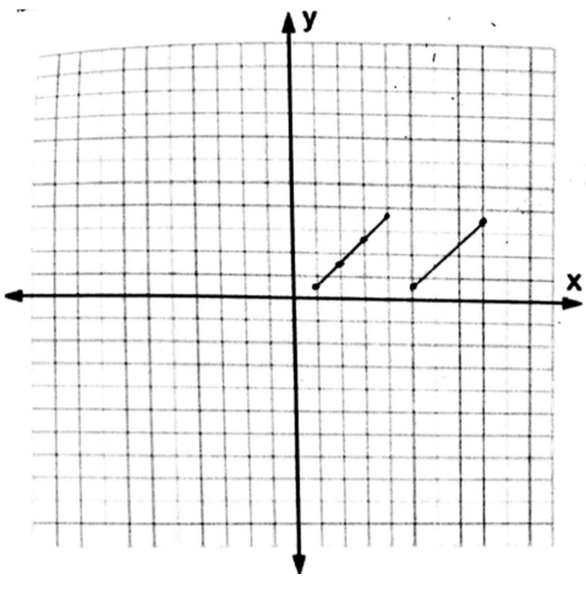

Figure 5. *The Answer Given by S3*

On the other hand, it was seen that the other student just filled in the table and could not draw the graph. In the interview with this student, the researcher asked some questions about the situation.

Researcher: "Although you have shared data as a group, you and a friend created the table and graph in different ways? Why did you prefer this?"

S4: "Actually, we are aware of the graph we want to create, but since we could not ensure that the walking speed of our two friends in our walk was equal, our graphs proceeded to intersect at one point rather than progressing parallel to each other. Starting was okay, but there was a constant speed problem, so I wanted to write like this."

In this question, it was determined that there are differences between the line graphs obtained after the walking instructions for the horizontal transformation given to the students and the expected ones. When the drawings of the two students were examined, it was seen that the line obtained after the walk and the given line intersected at one point. These students drew the data they obtained after the walk, but it was noticed that the other two students stated that there was a mistake in the graph and drew the lines which they thought as the correct lines indistinctly (see figure 6 and 7 below).

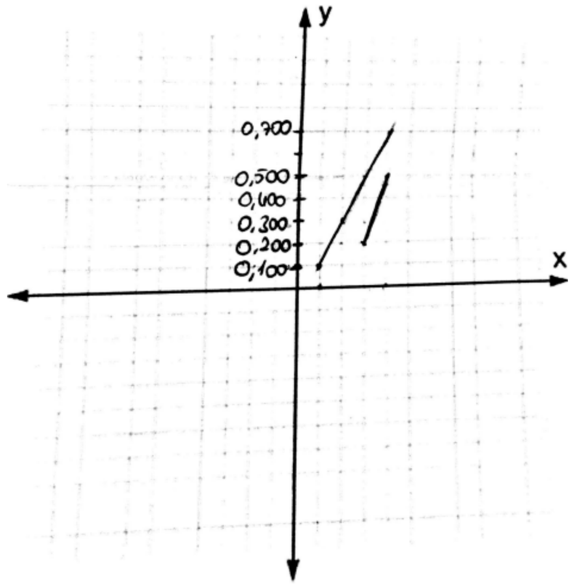

Figure 6. *The Answer Given by S1*

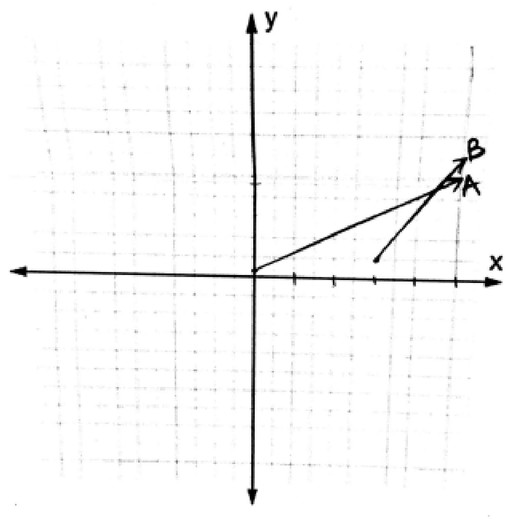

Figure 7. *The Answer Given by S2*

In this stage, like the questions in the last part of the vertical transformation, they were expected to answer the questions which were posed through a different line equation on the basis of what they had done during the activity and the knowledge they had discovered during the activity, but the students left this section blank by stating that they could not find a common result in the previous section. In other words, in this stage, the students were expected to perform horizontal transformation with the walking instructions and moreover to explain the obtained line graphs by using transformation terms. In this stage, they had more difficulties compared to vertical transformation. Some students tried to get the correct graphics they created in their minds by going beyond the data obtained through cooperation.

3.4. Persistence in Mental Associations

In this activity, it was considered important for the students to associate linear function with daily life and to establish mental associations regarding the concept of transformation. The interviews were after 5 months of the main study. It was aimed to examine the persistence of the concepts in the mind of the student. In these interviews, questions about vertical and horizontal transformation were posed. In these questions, the students were given an equation and they were expected to apply horizontal and vertical transformation operations. In addition, associations with daily life were also sought in the answers of the students. They were expected to remember the definitions and mathematical expressions of the concepts that they had previously discovered and about which they were expected to establish mental associations.

A student who could not answer the questions asked in the vertical transformation and horizontal transformation sections before and a student who did not have difficulty in answering the questions in the activity and who thought that he/she was good at technology gave the expected answers to all the questions asked during these interviews. It was also determined that these students made detailed explanations in their answers by establishing a relationship with the walk. Although these students could not obtain a graph as a group in the cooperation stage in the horizontal transformation section of the activity, it was determined that there were two students who drew the graphs on the basis of their own estimations. It was observed that the other two students could not establish mental associations. In order to find out the reason why the student, who could not find an answer to the questions in the activity, found answers in these interviews, another interview was held with the student and the following dialogue occurred between the researcher and the student during this interview.

> *Researcher: You looked at the question from a different perspective. What is the difference here?*
>
> *S3: In the beginning, I had just encountered the device. As I was a little tired and excited, I couldn't concentrate right away. Then I realized what I had to do. While answering the questions, I imagined the walking activities we did in the event and how I could do it in daily life. I answered the questions accordingly.*

The student realized the discoveries about transformation by visualizing them in his/her own mind. It was also seen that the student's experiences, fatigue, excitement, in short, his/her physical and mental state affected the process. The answers for the questions "*Take the function y=2(x+1). a. What would the motion that would yield this function be?" "b. If the function 2(x+1) were translated by 2 units towards right, what would the new motion be?"* are given by S3 like this, consecutively: "*a. There is a linear motion here with the constant speed, since for x=0, y=2, it should be 2 meters away from the starting point." "b. No change is observed in* *its speed of the walking. The new x=0 point should be located at 2 units before the previous version of the (0,0).*" It is seen that the student, who stated that he/she felt tired and excited, answered the question about vertical transformation with inferences from the situations that he/she observed in the activities conducted in the classroom and discovered by visualizing in his/her mind after the completion of the activities.

It is known that the student, who was interviewed for the second time after the interviews on persistence in mental associations, generally could not respond in individual studies related to horizontal transformation in the activity conducted in the classroom. It is also known that during the cooperation-based horizontal transformation activity in which he/she gave a faint answer to the question of drawing the right graph obtained from walking, unlike his/her group mates, who could not find a common answer.

The same student made his/her own discoveries individually in the interviews about persistence in mental associations and answered the questions about horizontal transformation as follows:

> *Take the function y=2x+5*
>
> *a. What would the motion that would yield this function be? Why?*
>
> *Answer: The function is a linear function and its slope is 2. I can say that there is a need for a motion to be performed at a constant speed. As y=5 for x=0, it should be 5 meters away from the starting point in the 0^{th} second.*
>
> *b. If the function y=2x+5 were translated by 3 units upwards, what would the new motion be? Please explain.*
>
> *Answer: As the slope does not change again, a linear motion at the constant speed should be performed. This time it should be 8 meters away from the starting point.*

Moreover, at the beginning of the study, the students were asked to rate their technological propensity out of 5. The purpose of scoring was to examine the effect and contribution of technological propensity in the process. As a result of the activity of persistence in mental associations, it was seen that the students with technological propensity score of 4 and 2 reached the correct answers. As a result of the interview conducted with the student having the propensity score of 2, it can be said that although the technology used is not used correctly, his/her active participation in the process and formation of ideas about the process in his/her mind contributed to the visualization of the walking situation in his/her mind. As a result of another study, it is stated that the use of a graphing calculator contributes to the ability of students to visualize function graphs in their minds, facilitates the association between graphical and algebraic representations and helps students to use visual strategies in solving function problems (Smart, 1995).

In other words, in this stage, the retention of the study, which was done about 5 months later, was checked with the questions directed to the students. It was understood from the answers and interviews that two students established mental associations. It was noticed that the other two students did not establish mental associations.

4. RESULTS AND DISCUSSION

The purpose of this study is to examine how the linear function and the transformation of the linear function on the axes are interpreted in the act of walking in daily life by university students using motion detectors and graphing calculators. The answers given by the students during the activity and the data obtained from the interviews conducted to check the persistence in mental associations were examined and evaluated.

In the examinations, it was determined that the students mostly performed the horizontal and vertical transformation of the function by selecting a point on the linear line and translating that point. It was observed that the students formed a line parallel to the given line, passing through the points they chose and carried, with equal slope angles. Different from this finding, Lage and Trigueros (2006) revealed that during the transformation process of the function, the students did the transformation by translating the entire function and did not express their opinion about the change of the points. However, each student in this study performed the transformation process according to a point they determined and they wanted to examine the changes of the points before the change of the entire function.

In another important finding of the study, it was observed that while some students could easily discover the function in the act of walking, they had difficulty in carrying the transformation of the discovered function to real life. Instead of interpreting the given function as an increasing position with respect to time, some of the students focused on the visual of the function and had the wrong judgment that it could be interpreted as height in real life. When using functions in the modelling of concrete situations of real life, it can be thought that there are connections between physical appearance and mathematical meanings. This situation is also expressed as topographic structures (Ural, 2006). There were times when the ups and downs observed in the position-time graph were considered by students as if they were ups and downs during a field walk (Monk, 1992). Therefore, the widely-held conviction that a topographic shape can affect the graphic drawing of the function and because misconceptions was also observed in the current study (Carlson, 1998; Monk, 1992; Monk & Nemirovsky, 1994).

According to the RME approach, which constitutes the theoretical background of the study, the problem situation should start with real-life situations that the individual can visualize in his/her mind. In this approach, it is argued that individuals should shape their real-life experiences in their minds and that this situation should be formulated and mathematized (Pınar, 2019). In the current study, a mathematical equation was given in the activity process and the students were expected to find its equivalent in daily life. The students were expected to discover the equivalent of the given mathematical equation in daily life, and then they were asked to make joint analyses between the mathematical expression they obtained and the daily life situation. The students could not immediately comment that the linear function state could be a constant speed walking, but they were able to uncover it after some observations, trials and peer discussions. Through different trials on what kind of walk it should be, they actually discovered linear functions with different coefficients. By shaping these discoveries with peer discussions and exchanges, they attained the desired walk. The starting point is questions that will enable the student to work with real-life situations. Although, the starting point here is not the real life situation, the RME framework is used since the idea of the formula is recreated in mind to make it real. These situations are based on didactic phenomenology, which is one of the important components of the RME approach, and they are designed in such a way as to allow the student to form mathematical knowledge on his/her own (Alacacı, 2016).

Learning through group work in which students have peer discussions is also one of the principles of RME (Alacacı, 2016). In the current study, both individual studies and collaboration are included. According to the data obtained, from time to time, students continued to seek answers to questions that they could not answer in individual examinations, in the collaboration phase. It was revealed in the interviews conducted to check the persistence of mental associations that the student had more permanent knowledge about the concept they learned first with his/her own efforts and after the discussions he/she made with the group by taking his/her own ideas into account. There are many research findings indicating that learning will be permanent and meaningful with discussions in the peer environment (Nakipoğlu, 2001; Çakır, Ballıel, & Sarıkaya, 2013). In addition, some students who could not reach a conclusion in the peer environment during the duration of the study were able to analyze and understand the relationship between walking and the function obtained by making self-evaluations. This showed that learning is an individual phenomenon as well as a social endeavour. Parallel to this finding, some educators believe that learning may not occur in social environments (Anderson, Reder, & Simon, 2000).

5. RECOMMENDATIONS

In the current study, it was realized that the relationship between daily life and mathematics should be examined bilaterally. It was observed that besides social learning environments, individual questionings are needed and that when students encounter a problem to be solved, they need to think about it for a while and to allocate some time to the understanding and interpretation of the problem. In light of the findings of the current study, it can be suggested that researchers conduct similar studies with other types of functions that can be examined in daily life by allocating longer time to group and individual work. Moreover, the idea of producing a relationship between daily life actions and mathematical concepts can help to examine mathematical modelling abilities of K-12 and university students.

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7. ABOUT THE AUTHORS

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Appendices

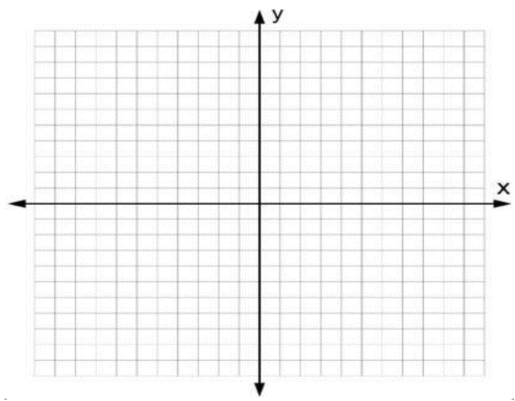
Appendix 1

APP-1 ACTIVITY

VERTICAL TRANSFORMATION

Stage 1

Draw a y=2x line on the coordinate plane.



Draw a line parallel to it, translating it 3 units up.

What is the equation of the new line? Explain how you found this line equation.

Check your drawings by making the new lines formed as a result of the operations you have done on graphing calculators.

Stage 2

What kind of walk do you need to perform in order to create the y=2x line given in the 1st question of Part A by using the motion detector? Please explain with its reasons.

In calculators that draw graphs using a single detector, try to construct the y=2x line by performing walks.

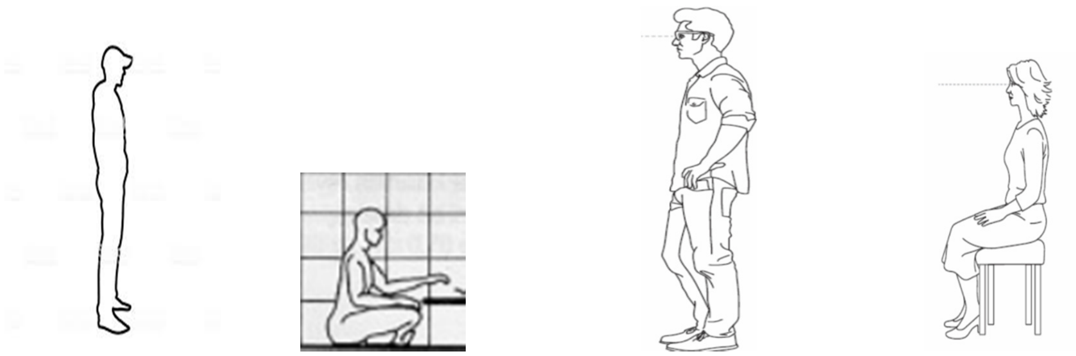
What kind of situations did you encounter during your experiments? Also discuss the wrong situations.

What kind of walk do you need to perform in order to create the line you found in the second question of Part A through transformation by using the motion detector? Please try this way of walking. Explain the results.

What kind of walk would you perform in order to form the new transformed line in graphing calculators by using a single detector? Please explain.

Stage 3

Organize your group as in the photo to collect data. Person D coordinates the collection of data. Person C walks. Persons A and B collect data.



A B C D

Persons A and B stand side by side. Each has motion detectors and graphing calculators attached to the detectors. Calculators are set to receive data from motion detectors for 5 seconds. A total of 5 data will be collected, 1 datum per second. Person C is opposite Persons A and B. In 0^{th} second, Person A starts to collect data from the detector by turning on the calculator. Person C starts walking forward from Person A. After 2 seconds, Person B turns on the calculator and starts collecting data from his/her detector. In 5^{th} second, Person A stops the detector. Person C continues walking, Person B continues collecting data. In the 7^{th} second, Person B stops collecting data and Person C stops.

We can briefly summarize the situation given above as follows:

0^{th} second: Person C starts walking slowly. Person A starts collecting data.

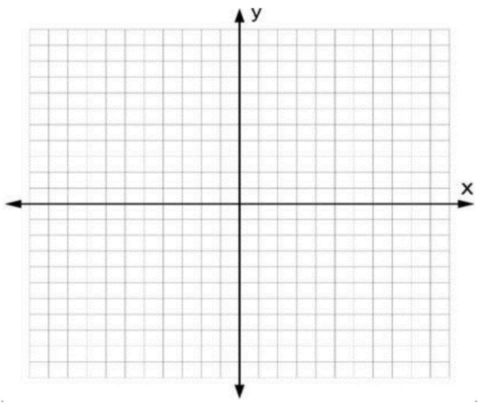
2^{nd} second: Person B starts collecting data.

5^{th} second: Person A finishes collecting data, stops it.

7^{th} second: Person B stops collecting data and person C stops.

Examine the data collected on the calculators. Fill in the table below with the data you obtained from the detector.

Time	Position of detector A	Position of detector B
1st second		
2nd second		
3rd second		
4th second		
5th second		



By using the data given above, draw the graph of the functions having these data on in the following coordinate system.

By using the two graphs formed and the table data and the concept of transformation as well, explain the relationship of the graphs with each other.

Let the graph created by person A be the y=f(x) function. In this case, how can the graph created by Person B be expressed in general?"

HORIZONTAL TRANSFORMATION

Stage 1

Draw a y=2x line on the coordinate plane.

Draw a line parallel to this, translating it 2 units to the right.

What is the equation of the new line? Explain how you found this line equation.

Check your new lines, which are formed as a result of the operations you have done, by making your drawings again on the drawing calculators.

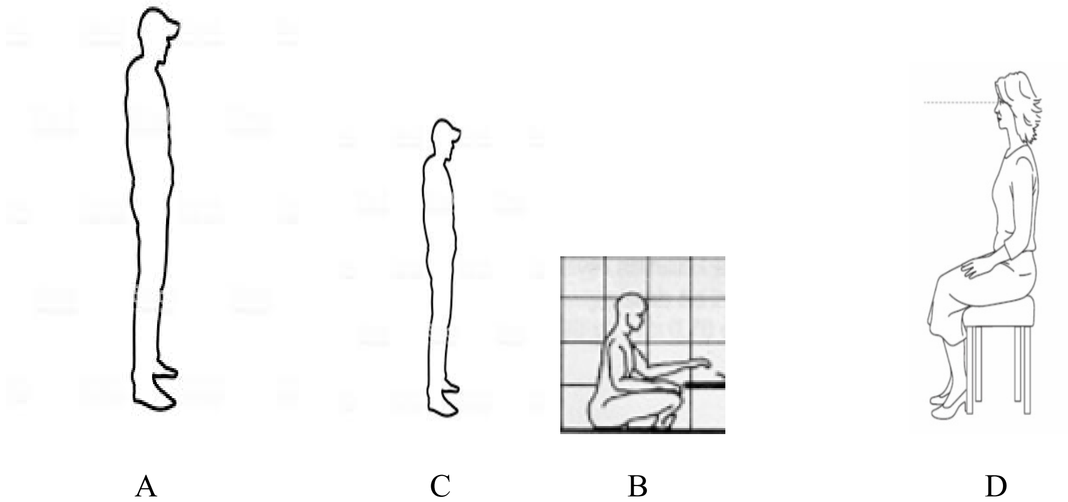
Stage 2

What kind of walk do you need to perform for the formation of the new line you found by transforming in the 2nd question in Part A by using the motion detector. Try this way of walking. Explain with its reasons.

What kind of walk would you perform in order to form the new transformed line in graphing calculators by using a single detector? Explain with its reasons.

Stage 3

Organize your group as in the photo to collect data.



Person D coordinates the collection of data. Person C walks. Persons A and B collect data.

Persons A and B stand in a single line, one behind the other.

Person B sits. Each has motion detectors and graphing calculators attached to the detectors. Calculators, motion detectors are set to 5 seconds. A total of 5 data will be collected, 1 datum per second. Person C is opposite Person A and behind Person B. People A and B run graphing calculators in 0th second, Person C starts walking towards Person D. After 2 seconds, Person C comes in front of Person B. In 5th second, Persons A and B stop collecting data with detectors. Person C stops.

The situation described above can be briefly summarized as follows:

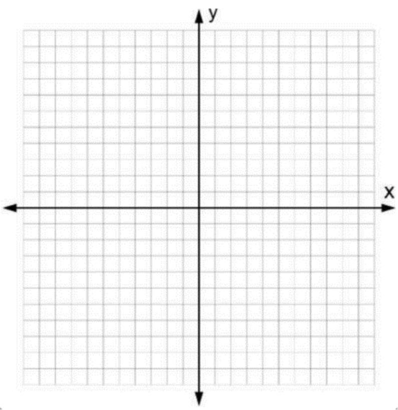
0th second: Person C in front of Person A starts walking slowly towards Person D. Persons A and B start collecting data.

2nd second: Person C comes in front of Person B after 2 seconds

5th second: Persons A and B stop collecting data, stop it.

Examine the data collected on the calculators. Fill in the table below with the data you obtained from the detector.

Time	Position of detector A	Position of detector B
1st second		
2nd second		
3rd second		
4th second		
5th second		



Using the data in the table, draw the graph of the functions that have these data on in this coordinate system below.

1. Explain the relationship of the graphs with each other using the two graphs formed and table data.
2. Let the graph formed by Person A be the function y=f(x). In this case, how would the graph created by Person B be expressed?

Appendix 2

Questions about Persistence in Mental Associations

Vertical transformation 2nd round interview questions

Question1: Take the function y=2x+5.

a. What would the motion that would yield this function be? Why?
b. If the function y=2x+5 were translated by 3 units upwards, what would be the new motion? Please explain.
c. What would the new function equation be like? Please explain.

Horizontal Transformation 2nd round interview questions

Question 2: Take the function y=2(x+1).

a. What would the motion that would yield this function be?
b. If the function 2(x+1) were translated by 2 units towards right, what would the new motion be?
c. What would the new function equation be like? Please explain.