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Implementation of the TPCK Model in the Manipulatives Designed by Teacher Candidates[1]

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Abstract: The current study examined the pre-service mathematics teachers' "manipulative material designs" and "the integration of designed materials into instructional design" processes in terms of technological-pedagogical content knowledge (TPCK). The research carried out within the "Material Design in Mathematics Education" course in the 2019-2020 Academic Year Spring Term covered 26 pre-service teachers. The research was an internal case study based on a qualitative paradigm. Data collection tools were video recordings, interview forms, and instruction methods where pre-service teachers integrated the materials they produced. The obtained data were analyzed descriptively according to the components in the TPCK Model. At the end of the research, the pre-service teachers stated that adequate discussion of the "conceptual infrastructure for the relevant acquisition" and "teaching methodology" would provide a more objective and error-free manipulative material and concept construction for students. In addition, other findings were that teachers' understanding of their relationship with the curriculum facilitated the process and that their review of their own learning positively affected the whole of instructional designs (aim, purpose, method, evaluation).

Keywords: *TPCK, Manipulatives, Instructional Design.*

1. INTRODUCTION

The most basic function of education is to prepare students for daily life and the business world of the future (Trilling & Fadel, 2009). Hence, business and political leaders expect schools to develop students' "21st-century skills" such as problem-solving, critical thinking, communication, cooperation, and self-management (National Research Council [NCTM], 2012).

Factors such as developments in the 21st century, improved information technologies, and globalization-related multicultural structures have changed the needs and expectations of societies, and education systems have also been affected by this change (Dağhan et al., 2017). The primary function of education in the 21st century is to educate individuals so that they can cope with the challenges of this century. For this reason, it is necessary to provide

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appropriate education to obtain problem-solving, questioning, investigative, communicative, productive, and collaborative qualified individuals (Ministry of National Education [MEB], 2013). This situation is closely related to science literacy, covering using scientific knowledge, defining problems, drawing evidence and human activity-based conclusions, and perceiving the world.

2. LITERATURE REVIEW

Mathematics, indispensable for daily life and all science branches, is also related to thought production, clear expression, systematic arrangement, and interpretation of scientific and daily life data (Fouze & Amit, 2017; Skemp, 1987; İnan, 2006; Umay, 2003;).

It is significant to integrate innovative technologies into the learning-teaching environment as much as possible in the mathematics learning processes of individuals who are expected to be technology practitioners and to have competence in solving technology-related problems. Technology positively supports mathematics education by helping students consolidate their knowledge and learn recent information (Lobo da Costa, de Carvalho, & Campos, 2017; NCTM, 2000). Considering the educational reforms, Türkiye has redefined teacher qualifications and has prioritized importing today's technologies to the classroom (MoNE, 2013). At this point, teachers have immense responsibilities. TPCK, which deals with the correct integration of appropriate technologies into the educational environment, was proposed by Mishra and Koehler (2006) by the integration of technological knowledge into the Pedagogical Content Knowledge (PCK) model developed by Schulman (1986). In both models, teachers deliver engaging teaching content using the proper pedagogical approaches. However, TPCK focuses on structuring the teaching content by blending it with appropriate technologies and pedagogies, unlike PCK. Therefore, in recent years, PCK and TPCK have mainly been used to reshape teacher training programs in the education faculties of many USA and EU universities (Abell, 2008; Köhler & Mishra, 2008). However, MoNE reported that Turkish teachers' and teacher candidates' knowledge, skills, and attitudes about the effective use of technology in education were not at the desired level (MoNE, 2013). In this context, it is crucial to examine the approaches of teacher candidates to integrate the manipulative materials they designed into the teaching environment in the TPCK model. From this point of view, the research aimed to examine the integrations of the manipulatives created by the primary school mathematics teacher candidates into the instructional design by associating them with the Technology Pedagogy Content Model, which is one of the technology integration models. The following questions sought answers for this purpose:

1) How do pre-service teachers handle (a) Technological Pedagogical Knowledge, (b) Pedagogical Content Knowledge, (c) Technological Content Knowledge, and (d) Technology Pedagogy Content Knowledge in their instructional designs in which they integrate the manipulatives they designed?

2) What points do pre-service teachers consider to determine their achievements while designing manipulatives that will be integrated into the TPCK-based teaching environments?

3) What are the pre-service teachers' opinions on the effect of in-depth analysis of content knowledge, technological knowledge, and pedagogical knowledge regarding their targeted achievement in integrating the designed manipulatives into the learning environment?

3. METHODOLOGY

This research, which examined the primary school mathematics teacher candidates' processes of creating manipulative materials and integrating them into instructional designs by associating them with the Technology Pedagogy Content Model, was carried out in a multi-case study pattern intertwined with a qualitative paradigm. In the multi-case study pattern, there is more than one case. However, each case handled or included in the research can be studied by dividing it into various sub-units. In this way, it is possible to compare situations (Şimşek, Yıldırım, 2008). The most distinctive feature of this design is that it allows for in-depth analysis by focusing on a current phenomenon, event, situation, individual, or group (Patton, 2014).

3.1. Participants of the Study

The current study, covering 26 third-grade teacher candidates studying in a state university's Elementary School Mathematics Education Department, employed the purposive sampling method to select participants. Purposive sampling is also called purposeful or judgmental sampling (Bernard, 2000). One of the purposive sampling methods is typical case sampling. All education faculties' similar material design courses in their curricula and the students' typical success in these courses in the past make this research sampling a "typical case." In addition, the study group consisted of third-grade students with less test anxiety than fourth-grade students but higher curriculum-related theoretical knowledge levels than first- and second-graders. In this context, the researcher aimed to reach pre-service teachers in the process of integrating technology/pedagogy/content knowledge based on the literature and to closely examine their situation in the process of putting theoretical knowledge into practice.

3.2. Research Process

The study, including 26 pre-service teachers, was conducted within the "Material Design in Mathematics Teaching" course. The research process is as follows:

1. Theoretical knowledge was structured by discussing "manipulative material design and development" in the context of "technology/pedagogy/knowledge integration" within the "Material Design in Mathematics Teaching course" for six weeks.

2. The pre-service teachers independently chose their group mates and achievements during this period.

3. Pre-service teachers entered an investigation process related to the chosen outcome. This process included revealing the mathematical definition of the outcome, its historical development, misconceptions/complexities/difficulties that students experience (or may experience), current teaching practices, and the handling of the outcome in textbooks.
4. The lecturer (researcher) conducting the course followed the pre-service teachers' Zoom sessions weekly and provided feedback.

5. In this process, pre-service teachers introduced the manipulatives designed and integrated them into their instructional designs.

The researcher analyzed the designs and, with the support of a field expert, revealed the conformity in subcategories and codes.

3.3. Data Collection Tools and Analysis of Data

Data collection tools in the study were the structured interview forms and the instructional designs into which the pre-service teachers integrated the developed manipulatives. Below are the structured interview questions addressed to the pre-service teachers:

- What did you consider while determining the outcome of the material design course?
- What was the impact of detailed mathematical structure analysis of the concept on the designed manipulative material during your manipulative material production process?

Instructional designs and structured interviews, in which the pre-service teachers integrated the created manipulatives, were analyzed descriptively under the components of TPCK. The descriptive analysis provides a summary and interpretation of the data via determined themes (Patton, 2018).

In the first stage of the descriptive analysis, a data analysis framework was created from the components of the TPCK, based on the conceptual content of the research. Accordingly, the data were analyzed under the sections of technological knowledge, pedagogical knowledge, content knowledge, technological pedagogical knowledge, technological content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge (Mishra & Koehler, 2006). These theoretical framework components are detailed below:

- Technology Knowledge (TK) covers all technologies from old technologies such as pen, paper, board, and chalk to modern digital technologies such as computer technologies, the internet, digital media, and the way of the use of these technologies (Koehler and Mishra, 2008; Mishra and Koehler, 2006).
- Pedagogical Knowledge (PK) refers to using knowledge and skills related to cognitive, social, and developmental learning theories, teaching methods, classroom management, lesson plans, assessment methods, and techniques at all teaching stages (Koehler & Mishra, 2008; Mishra & Koehler, 2006).

- Content Knowledge (CK) includes concepts, theories, ideas, organizational structure, evidence, and proof related to the subject area. It also covers approaches and practices to develop knowledge in a specific field (Koehler & Mishra, 2008; Mishra & Koehler, 2006).
- Technological Pedagogy Knowledge (TPK) includes the knowledge of which technologies can be used at which stages of the teaching process and how, considering the characteristics of technologic tools (Koehler & Mishra, 2008; Mishra & Koehler, 2006).
- Technological Content Knowledge (TCK) covers the selection, use, and evaluation of technology suitable for the subject area, contemplating the advantages of technology in effectively teaching students. This field of knowledge refers to having content knowledge that changes with technological applications (Koehler & Mishra, 2008; Mishra & Koehler, 2006).
- Pedagogical Content Knowledge (PCK) covers the teachers' knowledge and skills regarding the "teaching process" of the subject area. This knowledge area, generally the same as the approach proposed by Shulman (1986), refers to the effective teaching of a specific subject. This knowledge area can help to decide on which teaching method is appropriate for the content and how to structure and present the content according to student characteristics for successful teaching (Koehler & Mishra, 2008; Mishra & Koehler, 2006).
- Technological Pedagogical Content Knowledge (TPCK) is the knowledge area formed by the combination and intersection of technology, pedagogy, and content knowledge, which are the main components of the approach. It is generally defined as the combined knowledge needed to integrate technology into teaching in a particular subject area (Schmidt et al., 2009; Koehler & Mishra, 2008; Mishra & Koehler, 2006). This field of knowledge refers to effective technology utilization to increase the teaching quality at all stages, from the planning to the evaluation of the teaching process of a specific subject.

After a descriptive analysis of the pre-service teachers' designs according to the given theoretical framework, interviews were conducted to interpret the data set available. Another field expert has also evaluated the obtained data. The percentage of agreement among researchers was calculated using the Miles & Huberman (1994) reliability formula [Reliability = Agreement/(Agreement + Disagreement)]. The agreement rate among coders was 92% in instructional designs and 93% in structured interviews. The disagreement points between the coders appeared under TPCK and TCK, whereas an agreement and a consensus showed up in other coders.

4. FINDINGS

The four instructional designs produced by the pre-service teachers were examined in detail and shared in Table 1.

Table 1. *TPCK Applications in the Designs of Teacher Candidates*

TPCK applications in the designs of teacher candidates					
Designs	Number of steps in the design	Technological pedagogical knowledge	Pedagogical content knowledge	Technological content knowledge	Technological pedagogical content knowledge
Design 1	5	0	0	1	4
Design 2	9	0	0	5	4
Design 3	18	1	5	8	4

In designing environments where manipulatives were integrated, the teacher candidates were seen to focus on TCK and TPCK more. TPK was observed in only one design. An exemplary analysis of the design process is in Table 2 and 3 below.

Table 2. *An Example of A Construction Protocol*

An example of a construction protocol			
Construction Step	Process	GeoGebra Command	Motivation (Pedagogical-Content Support)
1	Adding the slider a	Select the slider tab (1,10), and enter the increment amount 1.	Defining the slider to a point and observing the features of the operation (the square formed with vertical edges) based on the line segment connected to this point. TPCK
2	Adding the slider b	Select the slider tab (1,10), and enter the increment amount 1.	Defining the slider to a point and observing the features of the operation (the square formed with vertical edges) based on the line segment connected to this point. TPCK
3	Adding a circle	A circle with a radius "b slider" is defined.	The circle with center A (r=b) makes the sliders relate to the areas of the squares on the vertical edges. TCK
4	Adding a vertical line	Select the perpendicular line tab and draw a vertical line to |AB|, passing through the center of the circle, and define the intersection point with the circle.	A right triangle is formed at the intersection of the perpendicular line and the circle. Squares are drawn on perpendicular sides. The square areas change with the movement of the sliders. These movements enable students to notice the changes in the fields. TPCK

Table 3. *An Example of A Construction Protocol*

An example of a construction protocol			
Construction Step	Process	Geogebra Command	Motivation (Pedagogical-Content Support)
1	Adding the slider a	Select the slider tab, and enter the Interval (-10,10) and Increment Amount 1.	Defining the slider to a point and observing the operation properties based on the vector connected to this point and the elevator picture. TPCK
2	Adding the slider b	Select the slider tab, and enter the Interval (-10,10) and Increment Amount 1.	Defining the slider to a point and observing the operation properties based on the vector connected to this point and the elevator picture. TPCK
3	Defining the vector u	The vector u=Vector ((-2.0),(-2,a)) is inputted.	Defining the addition operation of integers, based on the concept of direction in the elevator PCK
4	Defining the vector v	The vector v=Vector ((-4,a),(-4,a+b)) is inputted.	Defining the addition operation of integers, based on the concept of direction in the elevator PCK
5	Defining the point A.	A=(0,a+b) point is inputted.	Ensuring the operation is applied on the y-axis while performing the addition operation. CK
6	Identifying the line segment C	Line segment c= Segment ((-4, a+ b), A) is inputted.	The concept of direction helps to understand how is the process of addition. PCK
7	Identifying the line segment D	Line segment d=Line ((-4,a),(-2,a)) is inputted.	The concept of direction helps to understand how is the process of addition. PCK
8	Identifying the line segment E	Line segment e=Line ((-2.0),(0,0)) is inputted.	The concept of direction helps to understand how is the process of addition. PCK
9	Identifying the point B	The point B=(-2, a/2) is inputted.	It is the name of the vector indicating the direction and distance traveled by the "a" value on the line segments drawn. PCK
10	Identifying the point C	The point C=(-4,a+b/2) is inputted.	It is the name of the vector indicating the direction and distance traveled by the "b" value on the line segments drawn. PCK

11	Adding text	TextT3="("+a+")" is inputted.	The value of the number "a" is displayed. TK
12	Adding text	TextT4="("+b+")" is inputted.	The value of the number "b" is displayed. TK
13	Adding text	TextT1="+" is inputted.	The "addition sign" is displayed. TK
14	Adding text	TextT2= "=" is inputted.	The "equals sign" is displayed. TK
15	Adding text	Entering to the create text section through the slider section.	The result of the total is written. TK
16	Adding a picture	Click on the image option from the slider and add the downloaded image.	Helping embodiment by adding visuals TPK
17	Identifying the picture corners	Right-click on the picture. After selecting the properties, click the location tab in the box that opens on the right side. Define the corner points here.	Fixing the corners of the elevator picture and ensuring the integrity of the visual. TPCK
18	Identifying the point A_2	A_2 (0,a+b) is inputted.	The addition performed indicates the location of the elevator. TCK

Table 4 shows the analysis of the structured interviews conducted with the teacher candidates.

Table 4. *Themes and Codes*

Themes and codes		
Themes	Codes	Frequency
Technological knowledge	• Using technology effectively	K3, K4, K11, K13, K14, K26
Pedagogical knowledge	• Recognizing school-term features	K5, K7, K9
	• Connection with life	K19, K24
Content knowledge	• Recognizing the mathematical structure of the content	K1, K2, K3, K17, K21, K23, K24, K25
	• Knowing the way content is (was) handled in future and past school terms	K4, K5, K6, K8, K9, K10, K14, K16, K18, K20, K23, K24, K25
Technological pedagogical knowledge	• Visualization	K16, K18
Pedagogical content knowledge	• Daily life	K16, K17, K23
	• Concretization	K16, K17, K23
Technological content knowledge	• Knowing the technology	K10, K16, K18
Technological pedagogical content knowledge	• Correct integration	K7, K15, K22, K26
	• Holistic view	K7, K8, K11, K12, K14, K15, K19, K20, K21, K22, K24, K25, K26

Interviews with pre-service teachers revealed that they focused on selecting the most appropriate technical support for the content addressed in the learning outcome choice process. K15 stated, "I think I tried to select a relevant technology for my achievement. In other words, since it was basically (...) a material course (...), technology support was necessary and the most decisive element for success. Of course, I also had to make the required pedagogical emphasis on this technology. For this reason, I focused on the most visualizable and concrete gain."

Pre-service teachers should effectively use technology, which they will include in the learning outcome determination process. K26 stated, "... I think there is an application called Desmos regarding natural numbers. I wanted to use it in this task, but since I did not know everything about its features, I preferred to use GeoGebra."

A similar situation caused pre-service teachers to determine the gains from familiar subject areas. K20 explained this situation, saying, "Geometric shapes and their appearances are an area of my expertise. Therefore, the process of questioning the origin of the concept would also be easier. That's why I chose this topic."

K18, one of the two teacher candidates who drew attention to the importance of integrating content knowledge with appropriate pedagogy and technology, stated, “The detailed examination of the concept helped us decide on how to integrate the content into the technology and which pedagogical methods to select. Comprehending the concept, that is, knowing its mathematical structure, teaching methods, incomplete/wrong learning incentives, and preventive measures, ensured our improvement. Researching the mathematical background will help the researcher write the steps to follow in the manipulative designs more clearly and understandably. This perspective also grants us to see our shortcomings and mistakes.” K22, who shared the same view, said, “Questioning the history of the concepts, their mathematical background, their handling in the whole curriculum was very instructive to provide the opportunity to see practically what the integration of TPCK means. It was very instructive to provide the opportunity. Thereby, we could more clearly reveal the goals and targets the "learning gain" will provide.”

5. RESULTS AND DISCUSSION

In the education faculties of universities, it is imperative to focus on the quality education of the mathematics teacher candidates, who are the prospective experts who will give mathematics education to students. There are two primary goals in the mathematics education of pre-service teachers (Hiebert, Morris, Glass, 2003): (1) Ensuring they have mathematical field expertise and (2) developing their knowledge, skills, and tendencies toward "learning to teach" and giving them teaching skills. Today, teachers are expected to integrate technological knowledge also into this content and pedagogical knowledge perfectly (Mishra & Koehler, 2006). The current study revealed that the participant pre-service mathematics teachers, who were determining the manipulative materials that they would integrate into their instructional designs, focused on the learning gains that students could experience in their daily life and integrate with technology. In addition, pre-service teachers stated that questioning the mathematical background and focusing on appropriate teaching levels prevented their mistakes and shortcomings from affecting their manipulative design. In this context, the research revealed the importance of focusing on the mathematical backgrounds of the gains in the mathematics curriculum in training pre-service teachers. In addition, pre-service teachers stated that questioning the mathematical frame of the concept facilitated the manipulative design and the process management. This situation supports the finding of Richardson (2009) and Akkoç (2011) that "examination of the mathematical and educational background of the manipulative to be designed in the context of TPCK components will positively affect teacher candidates in "identifying student difficulties" and "finding solutions." From this point of view, pre-service teachers stated that questioning the mathematical background of the manipulatives allows them to record the designing stages more clearly and understandably. As a result, pre-service teachers should focus on the concept developments by considering the spiral curriculum in detail. This approach will help pre-service teachers consider situations in a prospective or retrospective manner, supporting pre-learning and enabling the detection of misconceptions in manipulative design, development, and application processes. Besides, it will prevent the concept from being designed in incorrect or incomplete contexts. In this context, pre-service teachers stated that integrating the model into the manipulative material made it possible to discover the goals and targets to achieve easily. This situation suggests the question Niess (2006) posed years ago: "What is the real issue? Is it lack of knowledge in incorporating technology into their mathematics education, or prejudices about how to teach mathematics using technology?" Literature studies on the subject show that pre-service and active teachers who are more equipped with technical knowledge, content knowledge, and pedagogical knowledge are more experienced in choosing the activities appropriate to the subject and are more experienced in incorporating technology (Harris and Hofer 2009, Özgün-Koca, Meagher & Edwards 2010). As a result of this situation, it can be suggested that TPCK Model components should be more focused on teacher training programs.

6. RECOMMENDATIONS

According to the study results, the researchers suggest that teacher candidates should be allowed to make TPCK-based designs and experience them throughout their education in different field courses. Moreover, teacher candidates should encounter various technologies and gain awareness of using alternatives. Finally, detailed research on how and to what extent the achievements in the mathematics learning process are taught at all teaching levels will significantly impact the materials that pre-service teachers will design, develop, and apply.

7. ABOUT THE AUTHOR

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8. References

Abell, S. K. (2008). Twenty years later: Does pedagogical content knowledge remain a useful idea?. *International Journal of Science Education, 30*(10), 1405-1416. https://www.tandfonline.com/doi/full/10.1080/09500690802187041

Akkoç, H. (2011). Investigating the development of prospective mathematics teachers' technological pedagogical content knowledge. *Research in Mathematics Education, 13*(1), 75-76.

Bernard, H. R. 2000. *Social research methods.* Thousand Oaks, CA: Sage.

Dağhan, G., Kibar, P. N., Çetin, N. M., Telli, E., & Akkoyunlu, B. (2017). Bilişim teknolojileri öğretmen adaylarının bakış açısından 21. yüzyıl öğrenen ve öğretmen özellikleri [21st century learners' and teachers' characteristics from ICT preservice teachers' perspectives]. *Educational Technology Theory and Practice, 7*(2), 215-235. https://dergipark.org.tr/tr/pub/etku/issue/30364/305062

Fouze, A. Q., & Amit, M. (2017). On the importance of an ethnomathematical curriculum in mathematics education. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(2), 561-567. https://www.ejmste.com/article/on-the-importance-of-an-ethnomathematical-curriculum-in-mathematics-education-5283

Harris, J., & Hofer, M. (2009). Instructional planning activity types as vehicles for curriculum based TPCK development. In Maddux, C. D. (Ed), *Research highlights in technology and teacher education*, (pp. 99–108). Chesapeake, VA: AACE.

Hiebert, J., Morris, A. K. ve Glass, B. (2003). Learning to learn to teach: An experiment model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6*, 201-222.

İnan, C. (2006). Matematik öğretiminde oluşturmacı yaklaşım uygulamasının örnekleri [Application samples of the constructivist approach in mathematics teaching]. *Journal of DU Ziya Gokalp Faculty of Education 6*, 40-50. https://dergipark.org.tr/tr/pub/zgefd/issue/47964/606856

Koehler, M. J., & Mishra, P. (2008). Introducing TPCK. Handbook of technological pedagogical content knowledge (TPCK) for educators, In Oliver M. (Ed.) *The AACTE Committee on Innovation and Technology Handbook of technological pedagogical content knowledge (TPCK) for educators* (pp. 2-29). https://doi.org/10.1080/17439884.2011.549829

Lobo da Costa, N. M., Carvalho, M. C. P. D., & Campos, T. M. M. (2017). Mediation of technological resources in lessons on polyhedra: Analysis of two teaching actions. In *Mathematics and Technology* (pp. 31-55). Springer, Cham. https://link.springer.com/chapter/10.1007/978-3-319-51380-5_3

Miles, M. B. & Huberman, A. M. (1994). *Qualitative data analysis: An expanded Sourcebook*, Sage.

Ministry of Education, (2013). *Secondary school mathematics (9th, 10th, 11th and 12th grades) curriculum*, Ankara.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054. https://www.learntechlib.org/p/99246/?nl=1

National Council of Teachers of Mathematics. (2000). *Principles and standards of school mathematics*. Reston, VA: Author.

National Research Council. (2012). *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas*. National Academies.

Niess, M. (2006). Preparing preservice teachers to teach mathematics with technology - developing a TPCK. In C. Crawford et al. (Eds.), *Proceedings of 18 Society for Information Technology and Teacher Education International Conference*, (pp. 3788-3795). Chesapeake, VA: AACE.

Özgün-Koca, S. A., Meagher, M., & Edwards, M. T. (2009-2010). Preservice teachers' emerging TPCK in a technology-rich methods class. *Mathematics Educator, 19*(2), 10-20.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage.

Richardson, S. (2009). Mathematics teachers' development, exploration, and advancement of technological pedagogical content knowledge in the teaching and learning of algebra. *Contemporary Issues in Technology and Teacher Education, 9*(2), 117–130.

Shulman, L.S. (1996) Paradigms and research programs in the study of teaching: A contemporary perspective. In: Wittrock, M.C., Ed., *Handbook of Research on Teaching*, 3rd Edition, Macmillan, New York, 3-36.

Skemp, R.R. [1987] *The psychology of learning mathematics*. (Expanded American edition). Hillsdalc, N.J.: lawrence Erlbaum Associa.

Trilling, B., & Fadel, C. (2009). *21st century skills: Learning for life in our times*. John Wiley & Sons.

Umay, A. (2003). Matematiksel muhakeme yeteneği [Mathematical reasoning ability]. *Hacettepe University Journal of Education, 24*, 234-243. http://www.efdergi.hacettepe.edu.tr/yonetim/icerik/makaleler/898-published.pdf

Yıldırım, A., & Şimşek, H. (2005). *Qualitative research methods in the social sciences*. Ankara: Seçkin.