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The Investigation of Cognitive Dissonances Experienced by Elementary Mathematics Teacher Candidates in Solving Non-Routine Problems

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Abstract: In this study, the opinions of teacher candidates about non-routine problems, the views about the strategies used to solve non-routine problems, and the accuracy of the decisions made for the solutions to non-routine problems were examined. The participants of this case study were 73 teacher candidates studying at the Department of Mathematics Education at a state university in the Eastern Black Sea region. The data were collected through questions consisting of non-routine problems and an open-ended questionnaire regarding these non-routine problems. The data obtained from the problems and forms were analyzed through content analysis. According to the results, when faced with a non-routine problem, some teacher candidates experienced cognitive dissonance in providing correct solutions while the others were simply not able to provide complete or correct solutions.

Keywords: *Cognitive Dissonance, Non-Routine Mathematics Problems, Mathematics Teacher Candidates, Primary Mathematics Education.*

1. INTRODUCTION

"Cognitive Dissonance Theory" is a state of psychological discomfort caused by the dissonance and inconsistency that occur in the minds of individuals during situations that require decision-making. It can also be described as the situation in which the individual is confronted with their emotions as a result of the presence of incompatible thoughts in their mind (Yücel & Çizel, 2018). The concept of cognitive dissonance was first introduced to the literature by the USA psychologist Leon Festinger in the 1950s (Aydın & Yılmaz, 2018). The concept of cognitive dissonance is the state of psychological discomfort experienced by the individual when it comes to choosing among various options (Festinger, 1957). Cognitive dissonance is the first theory that accepts and explains internal factors, namely emotions, which control the behavior that occurs when the information particles in the cognitive system do not match each other (Breker, 2009). The terms consistency or inconsistency in this theory represent the relationship between individuals, factors, and elements (Festinger, 1962). These elements refer to the cognitions acquired by individuals about their behaviors and the environment they live in. A few of these elements are what the individual wants and feels to

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do. Other pieces of information are also related to the environment in which the individual is living. Festinger (1957) mentioned that individuals' opinions are also information. Individuals do not form opinions about facts that are wrong according to them; therefore, when the opinions of individuals are examined in terms of psychology, there is no difference between opinion and knowledge. A similar situation encompasses the beliefs, attitudes, and values involved in cognition. All these opinions in the individual's mind are parts that make up cognition, and there may be consistency or inconsistency between them. Cognitive dissonance is the term that describes the dissonance between these parts. The process of cognitive dissonance is shown in Figure 1.

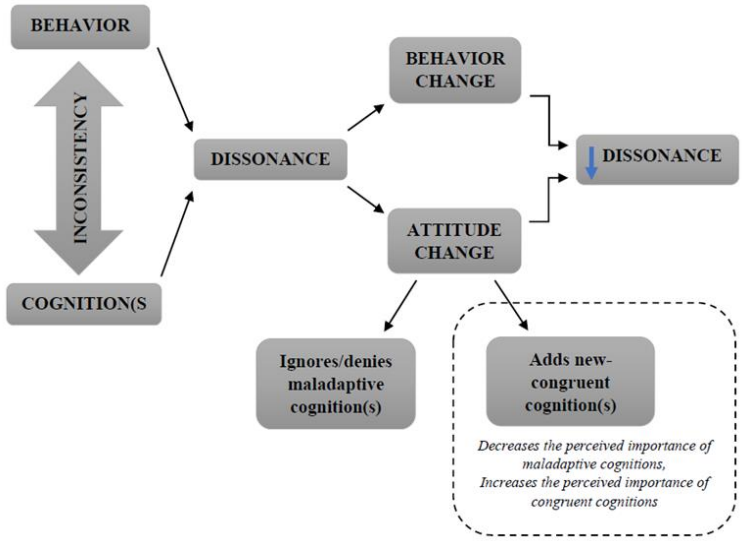

Figure 1. *Process of Cognitive Dissonance (Yücel & Çizel, 2018)*

One of the most important methods used in teaching mathematics is problem-solving (Taplin, 2006). Studies on various types of problems have been carried out so far. However, the basic distinction between these types is identified as being routine problems or non-routine problems (Aydın & Yazgan, 2018). While routine problems are seen as problems that can be solved with the knowledge of four operations; non-routine problems are defined as problems that cannot be solved by following an approach or a road map before solving the problem (Woodward et al., 2012). Examining research studies in the literature on problem types shows that classification of problem types has been made and this classification appears as routine and non-routine (Billstein, Libeskind, & Lott, 1993; Orton & Wain, 1994; Rappaport, 1966; Sovchik 1989).; Van De Walle, 2001). Skills that are important when solving non-routine problems are metacognitive skills such as classifying, associating, processing, and organizing data (Aydın & Yazgan, 2018). Each of these skills, which are necessary for solving non-routine problems, is shaped by the opinions of individuals, namely knowledge, and there is a process of dissonance in mind in order to reach the result of the problem. At this point, individuals can experience cognitive dissonance in solving non-routine problems.

Thanks to the studies carried out in the field of problem-solving, mathematics can be understood and applied more easily. By learning the steps of problem-solving, individuals

can produce various solutions for the situations they encounter in their daily lives, as well as in mathematics lessons, and tackle problems through these problem-solving methods. Problem-solving skills, which are one of the most common and basic skills in all known disciplines should be taught to students throughout their education life (Hacısalihoğlu Karadeniz, 2019). Routine problems are similar to each other with their similarity to previously solved problems or they require the application of a learned formula in a different problem (Polya, 1981). For the solutions to routine problems, the necessary ones from the four operations are determined and a solution is reached by doing them sequentially.

Situations that make individuals feel uncomfortable and uneasy may cause cognitive dissonance (Festinger, 1957). This feeling of discomfort and uneasiness may affect the plan followed in the solution of the given problem, as well as the decisions taken in the process, and the accuracy of these decisions (Soutar & Sweeney, 2003). In his Cognitive Dissonance Theory, Festinger revealed that cognition is a part of an individual's knowledge (Wisniarti & Sugiman, 2018). The reason for cognitive dissonance is the incompatibility of two cognitive elements in the mind (Cooper, 2007). To explain this with an example, continuing to smoke even though the individual knows that it is harmful causes cognitive dissonance. As a result of the relationship between these two conflicting thoughts, the individual may feel tense, uncomfortable, and uneasy. To overcome this, one of the thoughts of the individual is accepted in the brain by dominating the other thought according to that person’s way of thinking.

This research aims to examine the cognitive dissonance processes that teacher candidates encounter while solving non-routine problems and to reveal the cognitive dissonance they experience. In this regard, this study intended to reveal the cognitive dissonances that teacher candidates reflected on the questionnaire containing non-routine problems. In addition, the candidates' opinions on non-routine problems and the strategies they used for solving non-routine problems were examined.

2. METHODOLOGY

For this study, a case study methodology is adopted, which is one of the qualitative research approaches. Yin (1984), defines a case study as the study of an event or phenomenon in its natural environment by focusing on the questions of how and why. Thanks to the case study, it is possible to examine and investigate the reasons and the dimensions of a phenomenon that cannot be shaped by the researcher (Yıldırım & Şimşek, 2016). In other words, a case study is an approach that aims to describe a situation or an event occurring in a certain period through data collection tools such as interviews and observations and provide a detailed examination of the situation (Creswell, 2007).

2. 1. Research Participants

The study was carried out with teacher candidates who took the "Special Teaching Methods-II" course in the 6th semester of the 3rd year of mathematics teaching at a state university in the Eastern Black Sea region in the 2018-2019 academic year fall semester, through the

purposive sampling method. The reason why the participants were chosen from the third-year students was assumed that the students in this class had the necessary knowledge about problem-solving practices in the Special Teaching Methods I-II courses they took in the 5th and 6th semesters. Since it is known that almost all of the candidates were successful in the "Special Teaching Methods I" course they took in the 5th semester, the research began by accepting that they had the necessary knowledge about the problem-solving practices in the "Special Teaching Methods-II" course, which is the continuation of this course. The participants of the study consist of 73 teacher candidates. Out of 73, 54 of the participants are females and 19 are males. Participants of the study were coded as C_1, C_2, C_3, C_4, C_5, … C_{73} as required for the privacy of research and research ethics.

2. 2. Data Collection Tools

The problems used in the study were selected from the problems that appeared in the "National Mathematics Olympiad" held by TUBITAK in 2011-2013-2018 (TUBİTAK, 2022). Expert opinion was taken from two faculty members who worked in the field of mathematics education while selecting the problems. A total of four problems, consisting of one multiple-choice and three open-ended non-routine problems, were used. The reason a problem is multiple-choice is to limit the answer because the related problem has more than one answer. Due to the COVID-19 pandemic, the data collection tool was sent to the teacher candidates via e-mails, and they were given 3 days. Data were collected at the end of the period. While answering the questions prepared by the researcher, teacher candidates were asked to explain the problem situation and problem-solving steps as if they were explaining them to the students. They were asked to explain what they paid attention to in non-routine problem solving, how they solved the problem and the alternative solutions that came to their minds. Then, a questionnaire form prepared for the solution of non-routine problems was given to participants. After solving the questions containing non-routine problems, they were asked to answer the questions on the questionnaire about how they solved the problems, and to explain what they thought while solving them. There was no restriction on the answers so that participants could freely express their thoughts and solutions. Teacher candidates were given three days.

Below are the tables containing the questions used in the study and the thoughts and explanations regarding the solutions to the problems in these questions.

Table 1. *Non-Routine Problems Used in the Study*

No	Problems
P.1	The numbers 1,2, …, and 49 are written on the unit squares of a 7x7 checkerboard, with consecutive numbers in unit squares that share a common side. What is the maximum number of prime numbers in a row?
P.2	Nihal has a candy jar. Each time when she wants to get some candy, she takes half of the one more than the candies in the jar. If Nihal had to repeat this process five times to empty the jar, how many candies were in the jar initially? A)15 B)16 C)31 D)33 E)37
P.3	Let F be the farthest corner from corner A in a rectangular prism with a side length of 1, 2, and 3 cm. At least how many cm must an ant trip from corner A to corner F, provided that it always moves on the surface of the rectangular prism?
P.4	Ferries pass at equal time intervals from a pier, where the pier clock rings every hour. Watching the pier for a certain time on Sunday, Yusuf sees two ferries passing during this time and hears the clock ring once. On Sunday, Yusuf watches the pier for a longer period. Since Yusuf heard the clock ringing 16 times during this time, what is the minimum number of ferries he saw?

Table 2. *Questionnaires About the Solutions to Non-Routine Problems*

No	Purpose of the Question	Questions
1	Opinions of the participants about non-routine problems	1. Do you think the non-routine problems were difficult? 2. Was it easy to calculate the maximum number of primes occurring in a row? Please explain. 3. Was it easy to find the number of candies in the jar? Please explain. 4. Was it easy to find the distance between points A and F? Please explain. 5. Was it easy to calculate the number of ferries? Please explain.
2	Participants' views on strategies used to solve non-routine problems	1. How do you solve non-routine problems? 2. What formula do you use when solving non-routine problems? 3. Are the steps you used to calculate the maximum number of primes to be placed in the same steps as when calculating the candies in the jar? Please explain. 4. Were the steps of calculating the path of the ant going from corner A to corner F similar to the steps you tried to find the number of ferries Yusuf saw? Please explain. 5. What are the differences between these problems? Please explain. 6. Explain why you cannot follow the same steps when solving problems.
3	The accuracy of the decisions made regarding the solutions to non-routine problems	1. Did the strategies and formulas used in solving these problems lead you to the right solution? 2. In the checkerboard problem, did you try to place the numbers first, or did you calculate the maximum number of odd numbers in a row using the relationship between the squares? 3. Did you guess the answer before calculating the desired number in questions 2, 3, and 4? Explain how you guessed separately.

2.3. Data Analysis

Obtained solutions for the problems used in the study were classified as "correct" for complete and correct solutions and "incorrect" for incomplete, wrong, or incorrect solutions. The researcher classified problem solutions according to the solutions published by TUBITAK. In addition, different solution methods used by the participants were checked one by one by the researcher and were included in the classification. At the end of the classification, all analyzes were submitted to expert opinion, and corrections with reanalysis were made according to the feedback.

In the study, open-ended questions in the questionnaire prepared for the non-routine problems were analyzed by content analysis (Yıldırım & Şimşek, 2016). That is because, using content analysis, it is possible to gather similar data around concepts and themes with various characteristics and to present them to the individuals who regularly read them (Creswell, 2012). The data obtained were firstly examined with a holistic perspective, then the questions were classified according to their question numbers and examined in detail with the summative content analysis technique. In this way, a general view was obtained about the answers of the participants. Answers were coded carefully to avoid any researcher-based errors. It was questioned constantly so that what was sought in the data was not forgotten each time (Yıldırım & Şimşek, 2016). After the coding phase, similar codes were gathered around meaningful themes. After the themes were determined, sample answers were provided for each identified theme.

The data obtained from the questionnaire were detailed in the dimensions of the participants' views on non-routine problems, their views on the strategies used to solve non-routine problems, and the accuracy of the decisions made regarding the solutions to non-routine problems. Then, the questionnaires were analyzed by categorizing them according to whether cognitive dissonance occurred or not. At this point, the emotional-thought states of the teacher candidates while answering the questions, their expressions about themselves, and the explanations about the cognitive processes they experienced during problem-solving sessions were examined.

3. FINDINGS

In this section, firstly, findings obtained from solutions to non-routine problems are given below. Afterward, the results from non-routine problem solutions were presented.

"The numbers 1, 2, ..., 49 are written on the unit squares of a 7x7 checkerboard, with consecutive numbers in unit squares that share a common side. What is the maximum number of prime numbers in a row?"

Table 3. *Analysis of P.1*

Problem 1	f
Correct	35
Incorrect	38

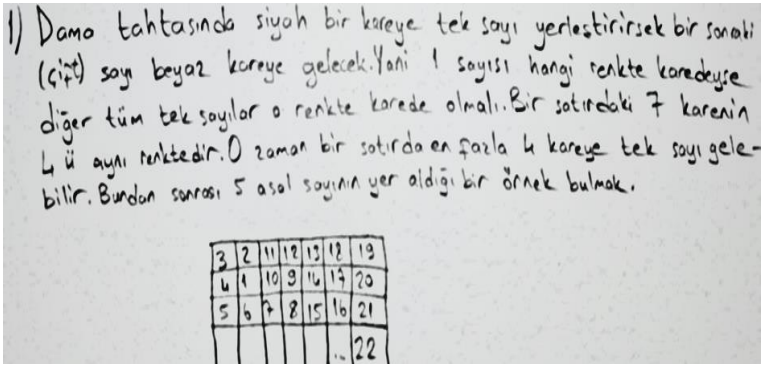

Figure 2. *C_{32}'s Correct Solution*

(In C_{32}'s solution, numbers are placed according to the board's black and white squares. C_{32} thought that if odd numbers were placed in black squares, even numbers will be placed in white squares. Eventually, 4 of 7 squares in a row will be of the same color. In that case, odd numbers can come up to 4 squares at most. After that, the thing to do is to find the row with 5 prime numbers.)

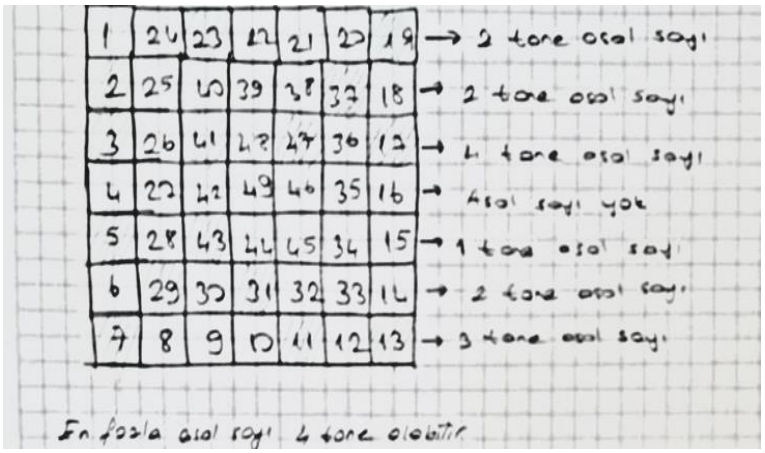

Figure 3. *C_{54}'s Incorrect Solution*

(In C_{54}'s solution, the numbers were placed in a spiral starting from the corner of the checkerboard and the prime numbers in the rows were calculated.)

"Nihal has a candy jar. Each time when she wants to get some candy, she takes half of the one more than the candies in the jar. If Nihal had to repeat this process five times to empty the jar, how many candies were in the jar initially?"

A)15 B)16 C)31 D)33 E)37

Table 4. *Analysis of P.2*

Problem 2	f
Correct	68
Incorrect or Incomplete	5

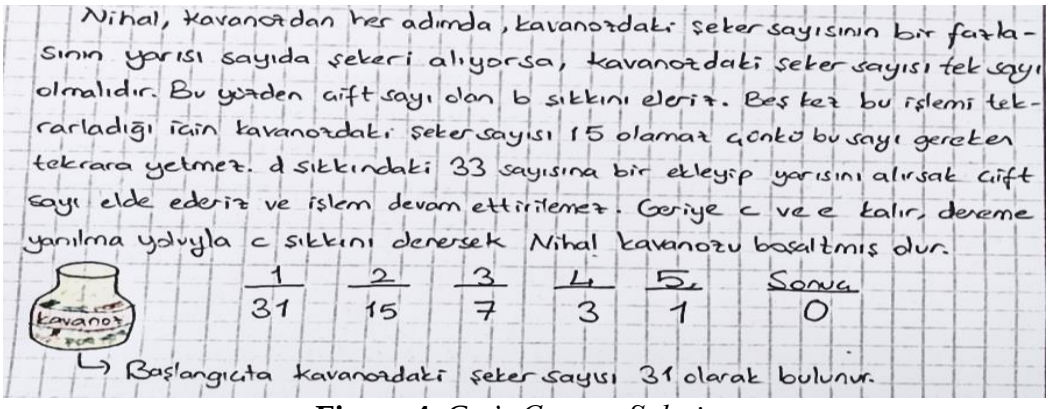

Figure 4. *C_{37}'s Correct Solution*

(C_{37} obtained the number of candy in the jar by using the numbers in the options. At this point, C_{37} explained that the number of candies in the jar could not be even, so C_{37} eliminated the numbers 15 and 33. C_{37} stated that they reached the result by trial and error strategy from the remaining options.)

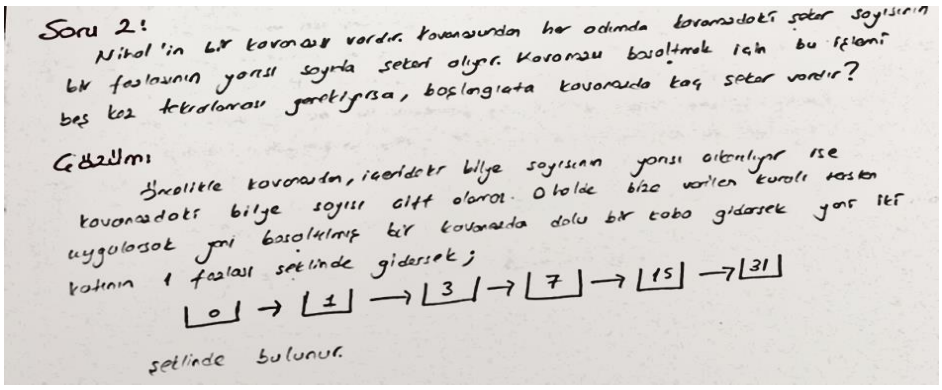

Figure 5. *C_{47}'s Incomplete Solution*

(C_{47} stated in their solution that there could not be an even number of candies in the jar and tried to solve the problem with the working backwards technique, but did not detail the solution method and the strategy they applied.)

"Let F be the farthest corner from corner A in a rectangular prism with a side length of 1, 2, and 3 cm. At least how many cm must an ant trip from corner A to corner F, provided that it always moves on the surface of the rectangular prism?"

Table 5. *Analysis of P.3*

Problem 3	f
Correct	28
Incorrect or Incomplete	45

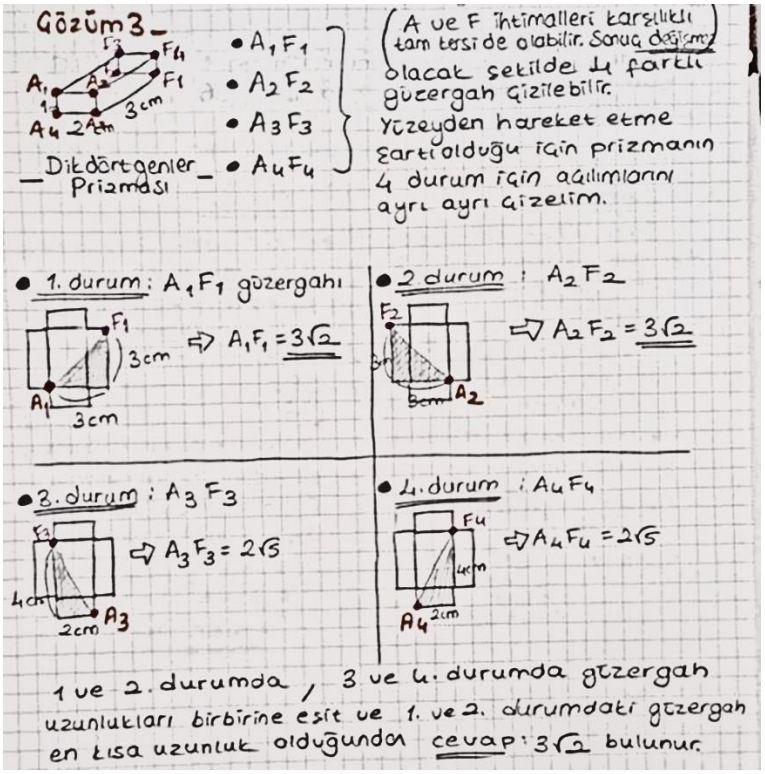

Figure 6. *C45's Correct Solution*

(In their solution, C45 explained in detail the path the ant will take and the length of the path by drawing the rectangular prism first in its closed state and then in its open state. At this point, they showed in their solution that the other routes to be followed by the ants moving from different corners will not change the path length to be traveled.)

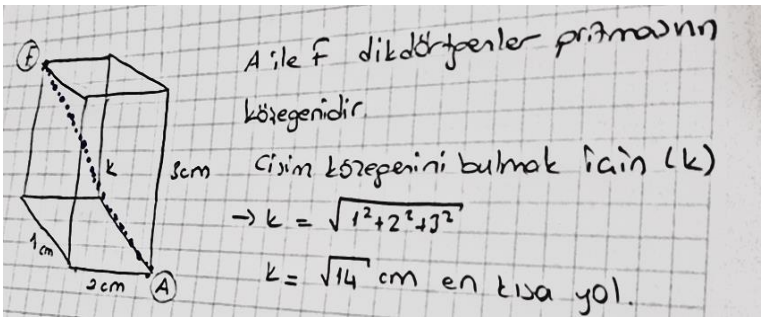

Figure 7. *C9's Incorrect Solution*

(C9 did not take into account that a prism is an object in their solution, so they moved the ant through the prism and calculated the distance between the corners.)

"Ferries pass at equal time intervals from a pier, where the pier clock rings every hour. Watching the pier for a certain time on Sunday, Yusuf sees two ferries passing during this time and hears the clock ring once. On Sunday, Yusuf watches the pier for a longer period. Since Yusuf heard the clock ringing 16 times during this time, what is the minimum number of ferries he saw?"

Table 6. *Analysis of P.4*

Problem 4	f
Correct	41
Incorrect or Incomplete	32

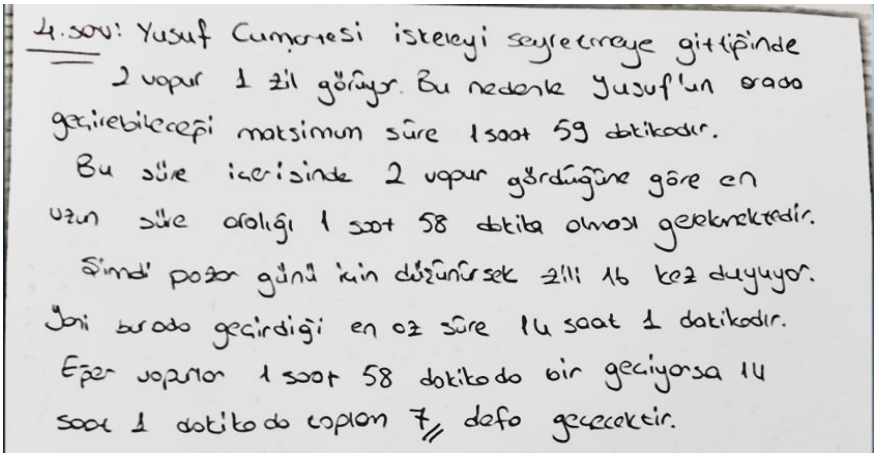

Figure 8. *C_{11}'s Correct Solution*

(C_{11} stated that Yusuf saw 2 ferries and heard 1 bell, and stated that the maximum time Yusuf could spend there would be 1 hour 59 minutes. They drew attention to the fact that the longest interval should be 1 hour and 58 minutes, as Yusuf saw 2 ferries during this time. C_{11} calculated that the minimum time Yusuf would spend would be 14 hours and 1 minute since Yusuf heard the bell 16 times on Sunday. C_{11} also stated that the ferries passed in 1 hour 58 minutes and reached the conclusion that 7 ferries would pass during 14 hours and 1 minute.)

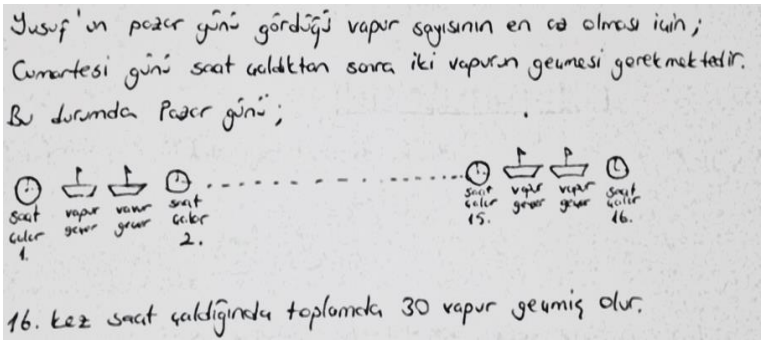

Figure 9. *C_{73}'s Incorrect Solution*

(C_{73} stated that for the number of ferries that Yusuf saw on Sunday to be the highest, two ferries should pass after the clock strikes on Saturday. Based on this, C_{73} thought that 2 ferries would pass every hour, and when the clock struck for the 16th time, a total of 30 ferries would have passed.)

Finally, findings obtained from questionnaires on non-routine problem solving are presented in the tables below.

Some of the answers given by the participants to the codes that emerged from the themes of "Thought-provoking" and "Time-consuming" in Table 7 are given below, respectively.

Table 7. *Teacher Candidates' Opinions About Question 1.1*

Question	Themes	Codes	f
1.1 Do you think the problems were difficult?	Time-consuming Thought-provoking	Easy	16
		Intermediate	36
		Hard	21

"I think the level of difficulty will vary with different grade levels. But when I look at it in general, I think that it is difficult to think about for a long time. They were problems that measured the level of reasoning rather than operational skill. Therefore, I think that students with advanced numerical intelligence will be able to answer these problems if they are given enough time. But it can seem complicated to different intelligence levels. It would be more appropriate for students to encounter these problems at later ages. So I would say it is difficult for the elementary mathematics curriculum. (Thought-provoking – C_{45})"

"The problems were problems that took some time. I think it might be a bit challenging for middle school students. (Time-consuming – C_{70})"

Eight of the participants, in the questionnaire form 1.1. made the following statements to explain the cognitive dissonance they experienced regarding the problem.

"I think the problems were difficult. That is because, although they seemed simple, they were quite distracting and confusing. I had to think a lot about it. (Thought-provoking – C_5)"

"The questions were thought-provoking. At first glance, it seemed easy, but then I concluded that it would be challenging, thought-provoking for a student, and could create discord in their knowledge. I think some questions require attention. I think it is necessary to practice such non-routine problems. (Thought-provoking – C_{12})"

"…while solving the problems, I did not conclude immediately because it takes a lot of time, so I reasoned. Even though I had difficulties because it didn't turn out right at first, I kept trying. (Time-consuming – C_{19})"

"I think the problems are difficult, not easy to do at first sight. First of all, it was necessary to understand the problem. When I understood the problem, I couldn't solve it because I thought my way about the solution is wrong. (Thought-provoking – C_{34})"

"When I looked at the problems, I thought they were difficult at first, so it seemed like I cannot solve them, but as I solved the problems, I saw that they were easy. We should not be biased toward problems; we need to think carefully. (Thought-provoking – C_{38})"

"When I first read problems, they seemed difficult. I thought I couldn't do it with the strategies I chose, but later on, I realized that they weren't difficult: you just needed to think carefully and I solved it. (Thought-provoking – C_{39})

"…The problems were so confusing that I am not sure whether I chose the correct path, I got nervous while solving them. I had a lot of doubts about the problems. (Thought-provoking – C_{51})"

"The important thing was to understand the problems well and find a mathematical connection, but I realized that I had difficulties even in these. (Thought-provoking – C_{64})"

Some of the answers given by the participants to the codes that emerged from the themes of "Importance of Understanding the Question", "Reasoning" and " Use of Problem Solving Strategies " in Table 8 are given below.

Table 8. *Teacher Candidates' Opinions About Question 1.2*

Question	Themes	Codes	F
1.2 Was it easy to calculate the maximum prime number occurring in a row?	Importance of Understanding the Question	Easy	31
		Intermediate	26
		Hard	16
Please explain.	Reasoning	It was a problem that required using logical reasoning	21
	Use of Problem Solving Strategies	It was a problem that required making a table or chart	13
		It was a problem that required guessing and checking	11
		It was a problem to find a pattern or use a rule	6
		It was a problem that required using the Sieve of Eratosthenes	4

"It was not very difficult to proceed by following the instructions given in the problem and to solve the problem by making use of the structure and features of the checkerboard. It just required some understanding of the problem. (Importance of Understanding the Question – C_3)"

"The problems were thought-provoking. It seemed easy to me at first glance, but then I came to the conclusion that it was challenging, thought-provoking for a student, and could create a contradiction in their knowledge…(Reasoning – C_{12})"

"I made a solution by drawing a figure in this question. If the figure is drawn and the numbers are placed correctly, anyone who knows what a prime number is can mark the prime numbers in the figure and find how many prime numbers are on the lines. (Use of Strategy – C_{33})"

Five of the participants made the following statements to explain the cognitive dissonance they experienced in answering problem 1.2.

"Actually, at first, I could not understand the problem properly and because the prime number part caught my attention, I couldn't understand that the board was a checkerboard. Therefore, I could not find a solution and it was difficult. (Importance of Understanding the Question – C_{17})"

"Our goal here was to get the maximum prime number together on the same row. We only need to be careful when placing odd and even numbers on the checkerboard. At first, I tried to solve the problems with the method I chose without paying attention to this, but I felt that something went wrong… (Importance of Understanding the Question – C_{26})

"When I first looked at the problem, I thought it would not be easy to solve and I was nervous while determining the steps to follow, but I thought that the colors on the checkerboard were black and white and they would come with odd and even numbers… (Reasoning – C_{39})"

"At first, I perceived it as a very simple problem and even though I found that there were 4 in my solution and I wrote my steps in the answer, I was not sure at all when I was doing it. When I discussed the question with my friends, most of them said 5; so, I tried to answer the question again in a different way… (Reasoning – C_{57})"

"It wasn't easy because it was difficult to sort the 49 numbers into 7 squares with the most prime numbers. This is because we have to play with numbers a lot and it was really difficult to think and write other rows while bringing the most prime numbers to the rows each time. Every time I tried, I couldn't help wondering whether what I did was right or not. (Use of Problem Solving Strategies – C_{62})"

Some of the answers given by the participants to the codes that emerged from the themes of "Features of Multiple-Choice Problems" and " Use of Problem Solving Strategies " in Table 9 are given below.

Table 9. *Teacher Candidates' Opinions About Question 1.3*

Question	Themes	Codes	f
1.3 Was it easy to find the number of candies in the jar? Please explain.	Features of Multiple-Choice Problems	Easy	55
		Intermediate	14
		Hard	3
		Obtaining the result through choices	17
	Use of Problem Solving Strategies	Ability to solve with strategy	14
		Using equations and algebra in strategy	8
		Ability to draw diagrams with strategy	2
		The importance of the determined strategy	23

"It was easy for me to find the number of sugars in the jar, but I think the part of the question was a little more difficult. If the student is not very familiar with the concept of the unknown, this question may confuse him, but explaining it in the same way as drawing can provide a clearer understanding of the solution of the problem. (Use of Problem Solving Strategies – C_5)"

"…I thought it could be solved by going through the options. The solution became simpler when I decided to reason about the solution in the operations of odd-even numbers. (Features of Multiple-Choice Problems – K2)"

Three of the participants made the following explanations to explain the cognitive dissonance they experienced regarding problem 1.3 in the questionnaire.

"First of all, I tried to solve the number of candies in the jar in a straight way. There were negative values or something, I said God, God. Then I read the question one more time and said that it is necessary to use a working backwards strategy here and continued that way. It was easy with the strategy. (Use of Problem Solving Strategies – C_{28})"

"When I was finding the number of candies in the jar, I thought it was a hard and challenging problem at first. However, later, by giving the value "x" to the unknown we learned, I carried out the operations according to the information in the problem and reached the result, although it was a bit difficult and I was not sure. (Use of Strategy –C_{31})"

"Now, as a mathematics teacher, I first tried to solve this problem using x, but I realized that it would be very difficult to explain the solution I made to the children using x. The children would be confused. Frankly, I had a hard time even doing it myself because I didn't quite understand what it was. Afterward, I understood the problem once and read it by understanding the logic and showed that the candy number in the jar could not be an even number, without being sure. If they asked me to find an answer without the options, I couldn't. (Features of Multiple-Choice Problems – C_{71})"

The participants' "Abstract Thinking Skill", " Properties of Geometric Objects ", "Spatial Thinking Skill", "Trial and Error", "Using Materials", " Information from Previous Experiences " and "Using Existing Formulas" in Table 10 of the participants. Some of the answers they gave for the resulting codes are given below.

Table 10. *Teacher Candidates' Opinions About Question 1.4*

Question	Themes	Codes	f
1.4 Was it easy to find the distance between the AF points?	Abstract Thinking Skill	Easy	42
		Intermediate	26
		Hard	5
Please explain.	Properties of Geometric Objects	Drawing the expansion of objects	26
		Drawing the shapes	12
	Spatial Thinking Skill	Enabling three-dimensional thinking	9
	Trial and Error	Try all possible situation solving	6
	Using Materials	Using similar objects	6
	Information from Previous Experiences	Solving a similar problem before	4
	Using Existing Formulas	Using the distance formula between two points	2

"The solution to this problem was also easy. But the important thing is not to be able to make that calculation; The main thing is to understand the logic until it comes to the account. To understand this logic and develop abstract thinking skills, there are plenty of figures, diagrams, graphics, tables, etc. It's important to draw things. By concretizing the problem with these methods, we understand it better and reach the solution more easily. (Abstract Thinking Skill – C_{36})"

"Since it is a three-dimensional object, I am a little confused in solving the problem, but in general I cannot say that it is a difficult problem. I solved it easily by drawing the unfolding version of the figure. (Properties of Geometric Objects – C_{16})"

"Three-dimensional shaped problems; I like the problem because it allows imagining on the figure and putting it on paper. Therefore, it was easy to find the distance since it was a problem style that I started with high self-confidence. (Spatial Thinking Skill – C_{15})"

"It was easy because after finding the farthest point, I saw the shortest distance with a few tries. (Trial and Error – C_{20})"

"…The problem becomes better understood when we try to solve the problem by thinking about the pill box or the matchbox, that is when we try to solve the problem by associating it with daily life. (Using Materials – C_{27})"

"Being prepared for the high school entrance exam and preparing for the university exam, I did not find it difficult to solve this problem because I solved different types of this problem and I knew the logic of this problem type. (Information from Previous Experiences – C_{60})"

"In this problem, it was important to know the formula for the distance between the diagonals of a rectangular prism and to put points A and F in the right place. (Using Existing Formulas – C_{9})"

Four of the participants made the following statements to explain the cognitive dissonance they experienced in solving problem 1.4.

"It seemed easy at first glance, but it turned out that I solved the question as a diagonal calculation with a wrong idea. After talking to my friends, when I read the question again, I realized that the ant was asking about the ant hovering on the surface of the rectangular prism, and I realized that I was doing it wrong and I thought of drawing the angle of the prism. After that, I saw that the AF length is the hypotenuse at the angle of the prism, so it helped me to draw a figure… (Properties of Geometric Objects – C_{12})"

"First of all, I thought that if I open the rectangular prism and choose the farthest corners with A and F open, I would find a smaller number and I did it this way, but when I heard the results from my friends, I thought I was wrong and changed my solution method. (Spatial Thinking Skill – C_{37})"

"At first, I did not know what to do, it was difficult, then I thought about how I could express and convey this to the student, and finally I chose to open the prism and show it there… (Using Materials – C_{57})"

"It is one of the problems that compels me. It took me 2 hours to understand and solve this question. It's a shame to say it, I thought I was stupid. I'm not sure about any of the steps I followed. But the real problem was with my math teacher in the past years. Because they couldn't teach us the skills to look in three dimensions. Let alone, if there was anyone who could see in three dimensions, otherwise, they would just say whatever and skip the subject. If I had teachers who came to class with the material in their hands, maybe I would not have had difficulty. (Using Materials – C_{64})"

Some of the answers given by the participants to the codes that emerged from the themes of "Reading Comprehension", "Reasoning", "Analysis of the Problem", "Use of Problem Solving Strategies" and "Use of Mathematical Language" in Table 11 are given below.

Table 11. *Teacher Candidates' Opinions About Question 1.5*

Question	Themes	Codes	f
1.5 Was it easy to calculate the number of ferries?	Reading Comprehension	Easy	4
		Intermediate	18
		Hard	51
Please explain.	Reasoning	Understanding the logic of the problem	20
		Calculating possible probabilities	11
		Establishing a relationship between the given	6
	Analysis of the Problem	Confusing and complex problem	21
	Use of Problem Solving Strategies	Determining a strategy for solving the problem	11
	Use of Mathematical Language	Turning the problem into a mathematical sentence	1

"I had a hard time finding the number of ferryboats. The reason was the uncertainty in the problem and the confusion in understanding the problem, so I can say that I had a hard time solving this problem. (Reading Comprehension – C_{29})"

"To solve the problem, I first tried to guess. But I couldn't get the text of the problem in my head without drawing a figure. What was given in the problem was also very confusing… (Reasoning – C_{34})"

"Calculating the ferry in the problem gave me a bit of a hard time because it asks for the least number of ferries. I tried many different solutions for the problem and found many different results… (Analysis of the Problem – C_{52})"

"I must state that I had difficulty calculating the number of ferries. Because I progressed step by step so that I could hear the clock chime 16 times from a certain time. I'd say it's a confusing problem. (Use of Problem Solving Strategies – C_6)"

"The number of ferries can be easily solved by writing a mathematical sentence and making use of the knowledge of ratio and proportion after understanding the problem. (Use of Mathematical Language – C_{46})"

Four of the participants made the following statements to explain the cognitive dissonance they experienced regarding the question.

"It was a problem I made after a long struggle. Although I was indecisive about the number of ferries he saw on the first day, I think that I have solved the problem. Usually, I have to wonder if there is something else in question such as least and most. (Reasoning – C_2)"

"It wasn't. You need to settle the problem in your mind and reach the correct solution based on your estimates. Therefore, it was difficult for me to find a mathematical connection. I do not know if the mathematical connections I've built make sense or are in line with what the problem is asking, but I did. (Reasoning – C_{15})"

"The problem of calculating the number of ferries frankly challenged me. I had several solutions and procedures in mind. As I read the question, my comments and thoughts increased. I tried to realize my solution by fitting the most logical of my thoughts to the question. (Reasoning – C_{31})"

"No, it was not easy for me because it was a question for me where mathematics conflicted with everyday life. I tried to exchange with full hours, but I thought that I could not get the result that the problem wanted, and I did my trades from scratch every time. (Analysis of the Problem – C_{72})"

Five of the answers given by the participants to the codes that emerged from the themes of "Problem Solving Strategies", "Importance of Understanding the Problem", "Problem Solving Steps", "Problem Situations" and "Reasoning" in Table 12 are given below.

Table 12. *Teacher Candidates' Opinions About Question 2.1*

Question	Themes	Codes	f
2.1 How do you solve non-routine problems?	Problem Solving Strategies	Determining the strategy to be used	40
	Problem Solving Steps	By determining the steps to follow	9
		Making a drawing about what is requested in the problem	7
		By following Polya's Problem Solving Steps	6
		Understanding the logic of the problem	5
		By determining the method for the problem	4
		By analyzing the problem	3
		Concretizing the problem	1
	Importance of Understanding the Problem	Understanding what the problem is trying to convey	20
	Problem Situations	By finding the relationships in the problem	6
		Associating the problem with daily life	4
	Reasoning	Reasoning	5
		Abstract thinking	1
		Guess	1
		Trial and error	1

"…If I need to visualize the question, I make it concrete by making it more comfortable to see everything in place. If he does not want to draw, I look at whether there is a solution strategy that the problem directs me to… (Problem Solving Strategies – C35)"

"As there are question types that certainly do not have a solution, which leads to more thinking, it is very important to first analyze what the question wanted to tell us since there is no specific formula or stereotyped understanding. And in this way, after understanding the question and following the remaining steps regularly, the result is easily achieved. (Importance of Understanding the Problem – C61)"

"I start solving non-routine problems by first understanding them well. I reason and focus on the solution by making judgments to find what is desired with what I understand. In my opinion, the solution to non-routine problems is best understood by understanding… (Problem Solving Steps – C62)"

"Since there are some unusual problems, when I read the question, I need to understand the question well and make a situation analysis. I'm trying to solve it with methods like attribution. (Problem Situations – C24)"

"Until now, the question types were almost always rote-based, so they could be easily solved by memorizing a few formulas. But unlike the current question types, it almost gives you the formula itself in the question. All that is required of you is to think abstractly and to find a logical solution. (Reasoning – C36)"

Four of the answers given by the participants to the codes that emerged from the themes of "No Solution to Problems with a Certain Formula" and "Mathematical Knowledge" in Table 13 are given below.

Table 13. *Teacher Candidates' Opinions About Question 2.2*

Question	Themes	Codes	f
2.2 What formula do you use when solving non-routine problems?	Problem Solving Strategies	Determining the strategy to be used	24
	Problem Solving Steps	Trying to understand the logic of the problem	11
		Trying the different methods to solve a problem	8
		Using diagrams and drawings	7
		By following Polya's Problem Solving Steps	7
		Interpret the problem	6
		By generating creative solutions to problems	2
		Associating the problem	1
	Reasoning	Reasoning	4
	Mathematical Knowledge	With the formulas on subject	1

"I don't use a formula, I determine the most appropriate strategy for the problem and I make a solution according to the strategy… (Problem Solving Strategies – K42)"

"It is not possible to use any formula when solving non-routine problems because these problems require thinking and doing different operations. That's why we can't use a certain formula. We can solve it by thinking in different ways. (Problem-solving steps – K67)"

"These problems can be solved practically, by understanding the question properly, by reasoning, and by using it in visual ways rather than operation. (Reasoning – K33)"

"When solving non-routine problems, I use whatever formulas are available under that topic and whatever is necessary. (Mathematical Knowledge – K23)"

Six of the answers given by the participants to the codes emerged from the themes of "Same Problem-Solving Strategies", "Different Problem Solving Strategies", "Differences Between Routine Problems and Non-routine Problems", "Use of Problem Solving Strategies", "Stages of Problem Solving" and "Reasoning" in Table 14 are given below.

Table 14. *Teacher Candidates' Opinions About Question 2.3*

Question	Themes	Codes	f
2.3 Are the steps you used to calculate the maximum number of primes to be placed in the same steps as when calculating the candies in the jar?	Same Problem Solving Strategies	Yes	4
	Reasoning	Partially	7
	Different Problem Solving Strategies	No	62
	Differences Between Routine and Non-routine Problems		
Please explain.	Use of Problem Solving Strategies	Using diagrams and drawings	16
		Working backward	13
		Using tables	5
		Using the concept of the unknown	5
		Calculate all probabilities	5
		Using equations	4
		Finding a pattern	3
	Stages of Problem Solving	Understanding the logic of the problem	12
		Determining the steps to be taken in solving the problem	12
		Using the features of the checkerboard	3
	Reasoning	Reaching results by trial and error	6
		Reaching the result by proceeding from the options	8

"We followed the same steps and calculated how many odd numbers should be placed in a row since there are 7 squares in the first problem. In the other problem, we decided that the total number of candies in the jar should be odd since half the sugar is given and all the candies in the jar must be finished in 5 steps. Then we performed operations on odd numbers. (Same Problem-Solving Strategies – C_{71})"

"Both had correlation findings. But I did some backward work on the jarring problem. I also used the relationship finding method when calculating how to place prime numbers. (Reasoning – C_{14})"

"No, it's different. Because these are different types of problems. You can't hunt rabbits with a fishing rod. Just as different capture techniques are used for different creatures, different ways are used for different types of problems. (Different Problem Solving Strategies – C_1)"

"These two problems were very different for me. Calculating the candies in the jar was more like a routine problem while figuring out how many prime numbers were non-routine problems. (The Differences Between Routine and Non-routine Problems – C_{23})"

"It's not the same, while solving the prime number problem, I solved it by drawing a figure, but in the problem of the candies in the jar, I started from the options. (Use of Problem Solving Strategies – C_{21})"

"In both problems, I first tried to understand the problem, and then I made a plan to solve these problems. As the third step, I implemented this plan. Finally, I checked its accuracy… (Stages of Problem Solving – C_{56})

Five of the answers given by the participants to the codes emerged from the themes of "Same Problem-Solving Strategies", "Reasoning", "Different Problem-Solving Strategies", "Using Problem Solving Strategies", "Stages of Problem Solving" and "Reasoning" in Table 15 are given below.

Table 15. *Teacher Candidates' Opinions About Question 2.4*

Question	Themes	Codes	f
2.4 Were the steps you followed in calculating the path of the ant going from corner A to corner F similar to the steps you tried to find the number of ferries Yusuf saw?	Same Problem Solving Strategies	Yes	6
	Reasoning	Partially	17
	Different Problem Solving Strategies	No	50
Please explain.	Stages of Problem Solving	Understanding the logic of the problem	23
		Drawing a diagram for the problem	18
		Calculate all probabilities	7
		Finding the relationships between the given	6
		Calculating what is required in the problem	5
		Drawing the nets of the prism	4
	Reasoning	Reasoning	4
	Use of Problem Solving Strategies	Solving similar problems	3
		Making a list	2
		Creating a diagram	1
		Sorting	1

"It was pretty much the same. Because I tried to go different ways by trying. The two problems had different paths. I tried all the ways and reached the desired result. (Same Problem Solving Strategies – C_4)"

"In the problem where we found the ant's path, I first determined the farthest corner, expanded it, and then tried the edge lengths given to us and found the minimum distance. In the number of ferries, I determined the difference between the transit times of the ferries, based on Saturday, and determined the least number of ferries that passed the number of bells on Sunday. Therefore, I can say that their steps are similar. (Reasoning – C_{11})"

"No, it wasn't similar or the same. They were problems that were solved completely differently. I thought of different steps for both and tried to solve them accordingly. (Different Problem Solving Strategies – C_7)"

"Yes, it was similar because corner A and corner F will be the farthest, but this will be the shortest distance, and in the problem with the ferryboat, we decided that whenever the bell rings, fewer boats will pass. So it was important for both of them to understand the logic of the problem. (Stages of Problem Solving – C_5)"

"No, because the ant's path problem is a style that I am used to, I decided to draw the situations with the direct likelihood (from the teachers), but the ferry problem was very difficult for me at first, until I solved the problem of my cousin. In the ferry problem, we continued by considering 2 hours and one minute less than 1 hour and 59 minutes... (Use of Problem Solving Strategies – C_{68})"

Some of the answers given by the participants to the codes that emerged from the themes of "Problem Solving Strategies", "Problem Solving Steps", "Types of Problems", "Problem Situations", "Mathematical Process Skills" and "Mathematical Knowledge" in Table 16 are given below.

Table 16. *Teacher Candidates' Opinions About Question 2.5*

Question	Themes	Codes	f
2.5 What are the differences between these problems? Please explain.	Problem Solving Strategies	The diversity of the strategies used	17
		The difference in the methods used	9
	Problem Solving Steps	The difference in solutions	17
		The difference in steps followed	7
		The difference in the difficulty levels of the problems	3
	Problem Situations	Diversity of topics including problems	5
		The difference in demand	2
	Types of Problems	Difference between routine problems and non-routine problems	
	Mathematical Process Skills	Having a different logic in every problem	4
		Each problem requires a different skill	1
	Mathematical Knowledge	Differences in mathematical knowledge	1

"These problems were all non-routine problems. Therefore, I had to think of different strategies for each problem. I think that a single rule and a single formula cannot be used in such questions. (Problem Solving Strategies – C_{70})"

"These problems are solved by different steps. I followed different solution steps for each problem. Since the problem sentences are different, I solved the questions with the appropriate steps. (Problem Solving Steps – C_{38})"

"We can think of problems differently as routine problems and non-routine problems… (Types of Problems – C_{35})"

"The issues to which the problems belong are different. Changing the subjects has also changed what is asked and given in the problems. Therefore, the strategies or formulas used in solving problems have changed. (Problem Situations – C_{55})"

"Some of them were quite difficult, not the basic-type questions, but questions to be solved by reasoning. Question 2 and Question 3 required some more processing. (Mathematical Process Skills – C_{49})"

"These problems generally like logical questions. It differed in terms of students' readiness. (Mathematical Knowledge – C_{66})"

Some of the answers given by the participants to the codes that emerged from the themes of "Problem Solving Strategies", "Problem Solving Steps", "Problem Situations", "Mathematical Process Skills" and "Problem Types" in Table 17 are given below.

Table 17. *Teacher Candidates' Opinions About Question 2.6*

Question	Themes	Codes	f
2.6 Explain why you cannot follow the same steps in solving the problems.	Problem Solving Strategies	The difference in the strategies used	19
	Problem Solving Steps	Difference of solutions	11
		The diversity of ideas developed to solve problems	3
		The difference in steps followed	2
	Problem Situations	Different from what is given and requested in each problem	15
	Types of Problems	Difference between routine problems and non-routine problems	7
	Mathematical Process Skills	Each problem requires a different thinking skill	1

"When I try to solve these problems with the same steps, I would be making a mistake. If I try to solve it in this way, I cannot solve the problems. Because these problems can be solved with different strategies. (Problem Solving Strategies – C_{7})"

"I couldn't follow the same steps as each problem requires a different solution. (Problem Solving Steps – C_{16})"

"Since each question is asked or given is different, we are not likely to follow the same steps in every question. (Problem Situations – C_{39})"

"The solution of non-routine problems does not require a single way, like routine problems. These types of problems appeal to thinking skills and problem-solving skills. The perspective of solving the problem may differ from person to person. It is not possible to follow the same steps as I do not encounter problems consisting of a sequence of operations. (Types of Problems –C_{15})"

"All of them required a different thinking skill. (Mathematical Process Skills – C_{10})"

Some of the answers given by the participants to the codes that emerged from the " Problem Solving Strategies" and "Problem Solving Stages" themes in Table 18 are given below.

Table 18. *Teacher Candidates' Opinions About Question 3.1*

Question	Themes	Codes	f
3.1 Were the strategies and formulas correct for solving these problems?	Problem Solving Strategies	Yes	51
		Partially	20
	Problem Solving Stages	No	0

"I think I used the right steps, and we can try using another strategy or another formula to figure this out. (Problem Solving Strategies – C_{13})"

"Yes, it was true. The strategies I chose led me to the solution and I evaluated the solutions I made for each problem. Of course, there are different solutions to these problems. (Problem Solving Stages – C_{52})"

Five of the participants are in the questionnaire form 3.1. They made the following explanations to explain the cognitive dissonance they experienced regarding the question.

"At first, I made the wrong choices in the strategies I chose. I didn't realize this until I saw my friends' solutions. Although the conclusion I reached was correct, my choices were wrong. Of course, other strategies can also be used. I tried to implement whichever is appropriate. Not everyone can solve a problem in the same way. (Problem Solving Stages – C_4)"

"Although I am not entirely sure, I think strategies I chose led me to the conclusion, or even if they did not, they brought me closer. So, I think it's partially true. (Problem Solving Stages – C_{20})"

"I think the questions I have created are correct, but I am not sure if I did it right in the problems that seek an answer to the question 'maybe'. So that's how I think my strategies and formulas are right or wrong. (Problem Solving Strategies – C_{29})"

"I often used the guess and check strategy Apart from that, I used the working backward strategy, drawing a table and finding a pattern. I think I reached the right conclusion. But I'm not sure of the correct way to go. (Problem Solving Strategies – C_{56})"

"I think my solution for the problem was true. Because the other strategies I used for the solution were not suitable for the problem. So, I tried to solve it most conveniently. But I'm not so sure. (Problem Solving Strategies – C_{69})"

Some of the opinions of the teacher candidates regarding their answers to the question in Table 19 are given below:

Table 19. *Teacher Candidates' Opinions About Question 3.2.*

Question	Codes	f
3.2 In the checkerboard question, did you try to place the numbers first, or did you calculate the maximum number of odd numbers in a row using the relationship between the squares?	See the relationship → Placement of numbers (S→P)	33
	Those who try both ways	4
	Placement of numbers → See the relationship (P→S)	36

36 of the teacher candidates placed the numbers first and then noticed the relationship between numbers and tiles. Twenty-four of these 36 candidates later changed their approach and preferred to place the numbers by seeing the relationship. Candidates who changed their answers made explanations to explain the cognitive dissonance they experienced. These explanations are given below in order.

"First of all, I tried to write the numbers and create a sequence according to the question. I found an answer in this way, but when I compared it, I saw that it was not correct and I gave up on this answer. After some thought, I was able to realize the relationship between the frames. (P→S – C_1)"

"I placed the numbers first. But then I realized that I always get the same result. This was strange to me too. Then I saw that when I used odd numbers, I already found 4 prime numbers. For this, I used 2 years, which is an even number. (P→S – C_4)"

"I tried to place the numbers here at first, but when I thought about it in this way, I started to get very different results. It could be too much, and I decided that it wouldn't be the logical of it. Then I did the above solution by trying to make a connection between odd and even numbers and prime numbers. (P→S – C_5)"

"At first, I placed the numbers, but the result did not seem to come from there, then I thought about how I could reach the result more shortly and calculated the number of odd numbers. (P→S – C_{10})"

"First, I started placing the numbers, and then I thought it wasn't necessary because no action resulted. I also sorted the numbers, I saw that the maximum number of prime numbers that could come one after another was the first 7 terms, and I got the right answer. (P→S – C_{17})"

"I placed the first row from 1 to 7. Since I did not understand that it was a checkerboard, I thought it would be long and troublesome to write one by one, and it was not like it would conclude. Because the prime numbers in the rows again would be counted. When I read it later, I saw that it was a checkerboard and I tried to catch a clue by drawing a shape. But I placed the first row from 1 to 7. I've seen that there can be 3 or at most 4 odd numbers in each line. (P→S – C_{19})"

"When I read the first problem, I already thought of the logic that the most prime numbers are in the first place, and I already intuitively felt that the most prime numbers were in the row where the numbers I would put first. However, to be sure and to see it clearly, I then placed the numbers in the boxes as the question asked me to do. Then I thought that I could not reach the result with this and solved the problem by using the relationship between the squares. ($S \rightarrow P - C_{35}$)"

"I tried to place the first numbers, but I realized that I chose the wrong strategy because I did not achieve what was requested. Later I used the odd numbers and the white and black parts of the checkerboard. I found a solution to this relationship. ($S \rightarrow P - C_{41}$)"

"First came the placing of numbers. I thought it would be very confusing. Later, I looked at the checkerboard and tried to establish a relationship between the colors. In doing so, I created too many cases out of the relationship between numbers. It may be that I am missing among these. The relationship between squares may be more accurate. ($S \rightarrow P - C_{51}$)"

"I wanted to take the two of them together. I thought I would get the wrong answer by trying to do it separately. So I placed the numbers first but I didn't know what to do for the result and I panicked. This confused me a lot. I also associated the colors of the squares with the numbers. From there, the result came out easier. ($S \rightarrow P - C_{53}$)"

"At first I tried to place the numbers, but as I struggled to reach the result, I tried to understand the relationship between the squares by doing different analyzes and thinking differently. I tried but couldn't get any result. ($S \rightarrow P - C_{62}$)"

"First I placed the numbers, but when I felt that I could not solve there, I later tried to establish a relationship between them. Then I noticed that out of 7 squares, 4 of them are the same color. According to the result I reached, I tried to calculate the number of odd numbers. ($S \rightarrow P - C_{64}$)

"I thought it would make more sense to place the numbers on a checkerboard, but only up to a point. I lined up some of them to see if they were lined up like this, not the whole checkerboard. Yet it did not come to an end. So I did it by calculating the relationship between squares and numbers. ($S \rightarrow P - C_{70}$)"

Some of the opinions of the teacher candidates regarding their answers to the question in Table 20 are given below.

Table 20. *Teacher Candidates' Opinions About Question 3.3.*

Question	Codes	f
3.3 Did you guess the answer before calculating the desired number in Problems 2, 3, and 4?	Problem 2	50
	Problem 3	50
	Problem 4	52
	Not Guessed the Answer	8

"No, I didn't guess but tried to reason. Because in such problems, it is more important to be able to proceed on the right path rather than making guesses. (Not Guessed the Answer – C_1)"

"I didn't guess in the second and third questions. I reasoned about the questions and performed four operations. However, in solving the fourth question, I made use of many assumptions. (4^{th} Problem – C_3)"

"2nd.? I thought that the number of sugars we have for the question should be a single number… In question 3, I guessed that the result would be radical… In question 4, I couldn't make a guess beforehand. Because I had a very difficult time thinking. (2^{nd} and 3^{rd} Problems – C_5)"

"When I was at the beginning of these questions, I tried to make a strategy estimation instead of the first solution that came to my mind and to predict the solution with this strategy. (All Problems – C_6)"

4. RESULTS AND DISCUSSION

In this study, cognitive dissonances experienced by teacher candidates while solving non-routine problems were examined. For this, firstly, non-routine problems were defined and the solutions to the problems were classified as "correct" for complete and correct solutions and "incorrect" for incomplete, faulty, or incorrect solutions.

When the solutions of P.1 discussed in Table 3 were examined, it was seen that 35 (48%) of 73 teacher candidates solved the problem correctly, while the remaining 38 candidates (52%) solved the problem incompletely or incorrectly. 36 candidates' solutions (49%) P.1 were incorrect, and for this reason, the most incorrectly solved problem was determined as P.1. When the answers were examined, it was found that one of the most common mistakes (21) made by the participants was the incorrect placement of the numbers due to not understanding the problem. At this point, although teacher candidates did not fully understand the problem they read, they may have had problems in finding what is required in the problem because they did not give the necessary attention and focus to the problem. It was revealed that other mistakes (15) occurred because of following incorrect solution steps or not taking all the necessary steps for the solution. It was observed that 2 candidates could not complete solving the problem and left it unfinished.

When the solutions of P.2, given in Table 4, were examined, it was seen that 68 (93%) of 73 teacher candidates answered the problem correctly, while the remaining 5 teacher candidates' (7%) answers were incomplete or incorrect. When the answers are examined to find out why 5 teacher candidates ended up with incomplete or incorrect answers, it was seen that there were mainly two reasons: not being able to reach the desired result in the problem (3) and processing errors (2). It was concluded that 54 of the teacher candidates (74%) used distractors to find the result, while 14 candidates (19%) used equations and unknowns. It has

been observed that the teacher candidates used the code "Reaching the result by going from the options (Table 9)" 17 times in the solution to this problem. It is wondered why teacher candidates use sentences with choices instead of multiple-choice problems, or proceed from the choices instead of eliminating the distractors. Considering the education, they have received, the reason for the use of such unscientific strategies should be investigated further.

When the data of P.3 in Table 5 were analyzed, it was determined that 28 of 73 teacher candidates (38%) answered the problem correctly, while the remaining 45 candidates (62%) answered the problem incompletely or incorrectly. In this problem, 32 (44%) of the teacher candidates solved the problem incompletely, and therefore it was determined that P.3 was the most incomplete. The reason for the incomplete solutions was determined as not showing the expansions of the objects, not specifying the points A and F on the closed state of the objects, not specifying the ways to proceed, and deficiencies in the explanations. It was determined that the reasons for the incorrect solutions (3) were caused by the misunderstanding and misinterpretation of the problem. It has been revealed that teacher candidates have encountered problems similar to this problem in their previous lives and have mathematical knowledge about the solutions to similar problems. Thanks to the examination of problems with similar problem-solving steps, thanks to various resources (teachers, worksheets, etc.) during the training they received before, the problem with the highest number of correct answers was determined as P.3.

When the data of P.4 in Table 6 were analyzed, it was determined that 41 of the 73 teacher candidates (56%) answered the problem correctly, while the remaining 42 candidates (44%) answered the problem incompletely or incorrectly. In this problem, 29 (40%) of the teacher candidates answered the problem incorrectly, while three candidates solved the problem incompletely. The most common mistake (25) in the problem emerged as a result of the teacher candidates' misinterpretation of the relationship between the time intervals and the departure time of the ferry. The reason for this can be shown as the fact that similar problems have been solved less than other types and that problems involving hours are less used. The remaining four errors are operational errors and it is thought that teacher candidates do not provide the necessary attention and attention in the process of solving the problem. It was seen that the reasons for the missing solutions emerged as a result of not establishing the relationship between the departure time of the ferry and the time intervals.

Teacher candidates' views on non-routine problems, their views on strategies used to solve non-routine problems, and the correctness of decisions about solving non-routine problems were examined. Examinations for these three purposes will be examined in the following sections.

4.1. Views of Teacher Candidates on Non-Routine Problems

According to the findings, the first analyzed department contains the opinions of the teacher candidates about non-routine problems. Candidates had the opportunity to examine non-routine problems before solving them. After the non-routine problems were solved by the

teacher candidates, the teacher candidates answered a questionnaire with the questions in Table 2. It was revealed that eight teacher candidates thought that they were not difficult questions when they first examined the non-routine problems. The reason for these thoughts is seen as the fact that teacher candidates encounter various problems throughout their education life. They also stated that they determined the simple steps to be followed for the solution of the questions and applied these steps.

When the questionnaire forms of the teacher candidates were examined, it was revealed that they did not have any problems with the problems before solving the problem, but after solving the problems, they thought that the questions were at an average or difficult level because they had problems about various points such as the problems being thought-provoking, taking time and needing to be examined in detail. These two problems were chosen by the teacher candidates as the most difficult questions in the questionnaire as a result of the inability to fully explore the order in which the numbers should be placed on the board in the first problem and the inability to determine the time intervals of the ferry in the fourth problem. While the teacher candidates stated that the relationship between numbers and the board should be explored for the solution to the first problem, they mentioned that for the fourth problem, the situation of the time intervals with the ferry departure time could be evaluated and the result could be reached. While most of the teacher candidates find the result by trying the numbers given in the options in the solution of the second problem, the majority of those who reach the result by drawing the expansion of the figure in the third problem. According to the candidates, the reason why the second and third problems are simple compared to the others is that there are choices in the second problem and they have solved similar problems in their previous lives with the third problem.

Eight of the teacher candidates have two views on non-routine problems. The first opinion of the candidates emerged as a result of the ideas they gained when they saw the problems for the first time, and the various information in their minds was compatible with each other. This view is that the candidates think that the problems are not difficult. After the teacher candidates solve non-routine problems, the new view that emerges, that is, the view that the non-routine problems they encounter are difficult, starts to experience incompatibility with the first view, which creates an incompatibility. This indicates cognitive dissonance. Teacher candidates' cognitive dissonance emerged when they started solving problems that they thought were easy. There may be various reasons why teacher candidates have difficulties at this point. The reasons for this may be that the candidates do not encounter non-routine problems frequently, encounter non-routine problems only in the classroom environment, and only deal with this type of question in the course dimension. Teacher candidates experience cognitive dissonance as a result of the routine problems frequently used in the education system and the steps followed to solve these problems do not yield results in non-routine problems. The reason why teacher candidates experience cognitive dissonance regarding non-routine problems is the unknown in the problems, that is, the unpredictable nature of the solutions to non-routine problems. As a result of the incompatibility of the two views, the

teacher candidates rejected their first view that non-routine problems were easy and stated that the questions were difficult.

To reduce the incompatibilities faced by teacher candidates similar to the ones above, emphasis can be placed on solving non-routine problems. In addition, teacher candidates should be allowed to solve non-routine problems according to their strategies. Teacher candidates can gain experience with various non-routine problems by experiencing various solution methods and strategies.

4.2. Strategies Used in Non-Routine Problems

When the views of the teacher candidates on how non-routine problems are solved, it is concluded that they think that determining the strategies to be used is a priority step that should be applied to solve such problems. When the teacher candidates' approaches to the questions and the analysis of their thoughts on the questions were examined, the code of "Problem Solving Strategies" ranked first in frequency. The reason for this can be shown as the fact that they actively use strategies in the Teaching Methods and Techniques courses they take and that they encounter problems more thanks to this course. It is thought that they understand the importance of strategy setting and applying it, as they cover strategies in various ways in their lessons. The "Problem Solving Steps" code that follows this includes the steps used by teacher candidates concerning Polya's Problem Solving Steps. It was concluded that if the candidates encountered a non-routine problem, they first tried to "understand the logic of the problem" and made various assumptions to "determine the steps to be followed". During the implementation of these steps, they made various unscientific statements such as "Understanding the logic of the problem" and "following Polya's steps".

When the teacher candidates were asked whether the non-routine problems were the same in terms of the steps used, it was concluded that the majority of the teacher candidates (P.1 and P.2 = 62, P.3 and P.4 = 50) stated that the problems required different problem-solving strategies. The basic code common to candidates is “Reasoning”. The main reason for this is to establish logical relations in problems and aim to conclude in line with these relations. When the answers given by the candidates to the questions are examined, the main reason for the cognitive dissonance is that the first strategy determined when the questions containing non-routine problems are encountered cannot lead the question in the desired direction. Candidates who cannot reach the result create an alternative by making various processes and thoughts about the existence of another strategy. They end the cognitive dissonance they experience as a result of the alternative solution they have created to meet the demands of the problem. It was observed that the teacher candidates could not solve the problem with the strategy they determined, and they gave up on the strategy they chose because of the result they encountered, and preferred another strategy and applied it.

In addition, since the teacher candidates had three days to answer these questions within the scope of the research, various information transfers took place between the candidates. Since

the candidates saw the solutions and results of other candidates, they experienced some cognitive dissonance processes and changed their solutions accordingly.

4.3. Accuracy of Decisions Regarding the Solutions to Non-Routine Problems

The second cognitive dissonance analysis focused on the ways and strategies used by teacher candidates in solving non-routine problems. The non-routine problem-solving strategies applied were analyzed for each question. The analyzes of the strategies used by the teacher candidates regarding non-routine problems were made according to the questionnaire forms in which the teacher candidates expressed their views on non-routine problems. The data obtained are detailed as follows. Thirty-six of the teacher candidates could not justify the strategy they used or the ways they followed in solving the first problem. The reason for this is that they place numbers on the checkerboard in a way they do not know-how. 36 candidates who first placed the numbers and then tried to find the relationship could not reach the result. As a result of the thought that the ways they determined could not reach the result and the incompatibility of the thought that they could reach the result by the ways they determined, cognitive dissonance emerged in the candidates. They have also changed the strategies to be used and the paths they will follow to achieve the result. The reason for the cognitive dissonance that arises as a result of the inability of the ways and strategies determined by the teacher candidates to solve the problem is that the path to be followed to reach the desired is uncertain. It is seen that some of the candidates changed their answers due to this indecision they experienced due to the uncertainty of the path to be followed.

Likewise, teacher candidates explained in detail in the questionnaires why they changed their answers. In the light of the findings of the study, the following recommendations can be for further research:

The questions and questionnaire used in this study were sent to the candidate teachers via e-mail because of the COVID-19 pandemic, and the data were collected back after three days. All analyzes and examinations were obtained from the documents sent by the candidates. To obtain the data in more detail and as unchanged as possible, it is recommended that the teacher candidates answer questions containing non-routine problems in the classroom environment and then make a clinical interview. Accordingly, the first impressions obtained and the dissonance processes in the minds of the candidates can be revealed more clearly. Questions containing non-routine problems used in the study can be selected in more detail from a larger question pool. In this way, cognitive dissonances experienced by teacher candidates can be investigated in terms of different dimensions and topics. Apart from non-routine problems, studies can be conducted on various mathematical topics that may cause cognitive dissonance. In this way, it can be examined whether other subjects in mathematics also evoke cognitive dissonance.

5. ABOUT THE AUTHORS

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