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Teachers' Thoughts on the Role of Mathematical Communication in Special Education

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Abstract: This study aimed to investigate teachers' thoughts about the role of mathematical communication in special education. Data were collected through semi-structured interviews with ten special education teachers responsible for teaching students with severe disabilities from various school levels. Data analysis used open coding to identify thematically common patterns. Results showed that special education teachers' thoughts on mathematical communication fell into six themes: (i) communicating mathematical ideas through embodiment, (ii) communicating mathematical ideas while considering cognitive competency, (iii) listening, speaking, reading, or writing mathematical language, (iv) developing positive attitudes towards using mathematical language, (v) communicating mathematical ideas through repetitions, and (vi) using mathematical language in other disciplines. Teachers have recognized that students with special educational needs require more time and effort to reach their full potential because their modes of mathematical communication can be particularly varied. Therefore, they discussed the concept of mathematical communication in special education from different perspectives and presented a number of approaches to it. Implications for future research and pedagogical practice were also discussed.

Keywords: *Mathematical Communication, Special Education Teachers, Teachers' Thoughts, Mathematics Education.*

1. INTRODUCTION

Mathematical communication is an essential skill that makes students' mathematical thinking explicit while doing mathematics. It refers to the ability to organize mathematical ideas, express them coherently and clearly, understand the mathematical thinking and strategies of others, and use the language of mathematics accurately and effectively (National Council of Teachers of Mathematics [NCTM], 2000). A clear emphasis is placed on communication by the National Research Council in adaptive reasoning for mathematical proficiency, with the capacity to think logically, reflect, explain, and justify (National Research Council & Mathematics Learning Study Committee, 2001). Mathematical communication involves the process by which students can make sense of problem situations and present their

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explanations and rationales for the solution process (Organisation for Economic Co-operation and Development [OECD], 2013).

Mathematics can be challenging for all students, but understanding mathematical language and concepts can be even more challenging for students with special educational needs (SEN) (Shellard, 2004). Although some students come to school with adequate communication skills, some others may need help in this area (Smith, 2012). In classrooms where support, encouragement, and opportunities for verbal communication are provided, students learn to communicate mathematically (Whitin & Whitin, 2003). However, students with special needs may have more difficulty learning the language of mathematics or communicating confidently about mathematics (Rubenstein & Thompson, 2002). Thus, particular attention is given to supporting the use of mathematical language in the classrooms and to assessing students' communication (Owens, 2006). Teachers must help students communicate ideas by getting them to reflect on what they already know (Schmidt, 2004) and build a smooth connection between mathematics and language (Stigler & Hiebert, 2004). Ensuring the effective use of mathematical language in the learning environment is critical not only for the development of mathematical communication skills but also for making sense of mathematical concepts (Pourdavood & Wachira, 2015).

Considering the importance of developing students' mathematical communication skills, the responsibilities of teachers in this regard cannot be ignored. For example, Morgan (2016) emphasizes that teachers have a central position in the development of students' mathematical communication skills. The Ontario Ministry of Education (2020) identifies communication as one of the seven mathematical processes and indicates that students should learn to use it effectively. It is recommended that teachers be aware of the importance of communication skills and provide students with various opportunities to express and understand mathematical thinking and to participate in mathematical discussions using everyday language. Cobb et al. (1994) state that students cannot use the language of mathematics without support and that teachers should guide their students in using this language. Teachers have a key role to play in creating a classroom environment in which mathematical communication is strongly involved as an integral part of mathematics instruction (Kaya & Aydın, 2016). Hence, teachers' thoughts on mathematical communication and their responsibilities in developing it gain importance. Teachers' lack of awareness of this issue can negatively affect students' active participation in the instructional process and their success in mathematics (Staples & Truxaw, 2010). Developing mathematical language and sharing ideas with peers and teachers can increase mathematical achievement for all students, but especially for those with special needs (Ernst-Slavit & Slavit, 2007; Maccini et al., 2007). Therefore, it is of significance to examine what special education teachers think about mathematical communication in order to successfully promote equitable practices in mathematics instruction.

Moreover, the way teachers think, believe, and act in the classroom has a significant impact on young people's learning. Teachers do not just set the curriculum. They also develop it, define it, and reinterpret it. What teachers think, what they believe, and what they do in the classroom, therefore, ultimately shapes the kind of learning that students acquire (Hargreaves, 1994). There is a close relationship between the beliefs teachers hold about the teaching process and how they design their learning to achieve the intended goals (Pajares, 1992). Teachers' beliefs are the driving force or motive for their teaching because they influence their knowledge of the subject and curriculum as well as their knowledge of how students learn and how to teach mathematics (Thompson, 1992). On the other hand, teachers' thoughts may well be antecedents and creators of teachers' beliefs (Pajares, 1992). Moreover, Ernest (1988) found that among the many underlying factors that affect mathematics teaching practice, teachers' thoughts and their level of reflection are the most salient. Therefore, it is worthwhile to examine what teachers think about mathematical communication, as their thoughts have an important role in shaping their classroom dynamics and behaviors. Accordingly, the purpose of this study is to investigate special education teachers' thoughts about the role of mathematical communication in special education.

2. LITERATURE REVIEW AND CONCEPTUAL BACKGROUND

2.1. Mathematical Communication

Mathematical communication is a means of exchanging ideas and clarifying understanding (NCTM, 2000). Speaking, listening, reading, and writing are all essential forms of communication in mathematics classrooms (Ramirez & Celedón-Pattichis, 2012). Mathematical learning takes place in a linguistically sensitive social setting that considers linguistic requirements and discourse components (Chval & Khisty, 2009). This setting features teacher-supported, dynamic, comprehensive interactions that include all types of teacher-student and student-student communication (NCTM, 2020). Mathematical discourse involves the conscious exchange of ideas in dialogues and other forms of verbal, visual, and written communication (NCTM, 2014). According to Turner et al. (2015), communication skills include both receptive and productive components. The receptive component involves understanding what is being given, what information is important, and what type of response is expected in terms of the mathematical goals of the task. The constructive component focuses on the presentation of the answer including the solution steps, an explanation of the thought process used, and a rationale for the answer offered.

The importance of mathematical communication or the necessity of developing it is frequently stressed by many mathematics educators. Hoyles (1985) states that encouraging students to communicate mathematically not only helps them express their thinking, but also moves the class from a teacher-centered to a student-centered learning paradigm. This also improves students' conceptual understanding of mathematics. According to McLennan (2014), mathematical communication allows students to clarify and represent their understanding as well as discuss new problem-solving strategies. Kostos and Shin (2010) support that communicating mathematically contributes to the development of students' mathematical thinking and problem-solving skills. Jung and Reifel (2011) agree that communication is one of the most effective ways to improve students' conceptual understanding, problem-solving skills, mathematical reasoning and sense-making. Barwell

(2008) notes that learning and teaching mathematics is a more language-based process and believes that it is important for teachers and students to acquire mathematical communication skills in this process. Chard (2003) argues that understanding the language of mathematics provides students with the ability to think about and discuss mathematical concepts. Matteson (2006) contends that students' fluency with the language of mathematics helps them develop creative mathematical ideas. Martinez (2001) holds that communication is an important tool for students to effectively present their mathematical ideas and assess their learning. Kinman (2010) asserts that mathematical communication allows students to recognize their misconceptions and correct them. Communication in mathematics classrooms enables students to reflect on their thinking and learn from others through sharing ideas (Chapin et al., 2003) or to successfully transfer students' mathematical thinking to their peers and teachers (Cooke & Buchholz, 2005). Hence, mathematics education emphasizes the importance of classroom discourse in assisting students' conceptual development (Cobb et al., 1997). Getting students to focus on the linguistic aspects of mathematics can help them discover and better comprehend the meanings of mathematical concepts (O'Halloran, 2000). Since mathematical communication is a central component in teaching mathematical concepts and fostering mathematical thinking, establishing mathematical communication skills, including students' oral and written communication with their teachers or classmates, is an important goal of mathematics education (Pape et al., 2003). Moreover, as Patchan et al. (2022) argue, the ability to communicate ideas is not a simple goal in itself, but an essential component of developing an understanding of mathematical concepts, mastery of procedures and problem-solving, and a positive attitude toward learning mathematics.

2.2. Students with SEN and Mathematical Communication

Allowing students to verbalize their thoughts is also a practice that positively impacts the mathematics learning of students with SEN (Fuchs et al., 2017). Although students with learning disabilities achieve a higher level of understanding and gain important insights into mathematical principles through the process of verbalization and reflection, it seems unlikely to see teachers promoting verbalization in special education (Gersten et al., 2009). This is because teacher-directed algorithmic instruction where the teacher provides a step-by-step process for solving a particular type of problem before students use it to solve other problems of the same type is the predominant instructional paradigm for teaching students with disabilities (Jackson & Neel, 2006). However, due to a variety of unique features in their cognitive and communication skills that require appropriate attention, students with autism spectrum disorders have limited opportunities to benefit from formal schooling (Santos et al., 2015). Therefore, learners who have been identified as having special needs require particular support to improve their mathematical communication skills (Ontario Ministry of Education, 2006). Teachers need to encourage them to communicate so they can understand what they are thinking and then draw on this information to shape their instruction, promote understanding and reinforce learning (Ontario Ministry of Education, 2005). Teachers who stimulate social and dialogic interactions among students with SEN help them foster their language acquisition (Purcell-Gates et al., 2011) and mathematical understanding (Lambert et al., 2020; Lei et al., 2020; Stein et al., 2015). This would also enable them to communicate successfully with classmates and teachers, which is essential for academic learning and cognitive development (Ugalde et al., 2021).

Therefore, mathematical communication is considered extremely important for the development and promotion of mathematical skills of students with special education needs. On the other hand, teachers' thoughts on mathematical communication, which enable them to establish this communication in the classroom, are of great importance in their approach to this issue. Teachers' thoughts facilitate the determination of their beliefs and attitudes because their thoughts can be a precursor to their beliefs (Pajares, 1992). Hollingsworth (1989) found that how teachers implement new methods or programs in their classrooms is related to whether teachers' beliefs are consistent with the proposed new methods or programs. In addition, Ernest (1988) stated that teachers' thoughts are among the important factors that influence mathematics teaching practices. Thus, it is important to investigate what teachers think about mathematical communication because their thoughts play an essential role in shaping their practice. Accordingly, this study aims to explore special education teachers' thoughts about the role of mathematical communication in special education. The following research question guides the study: What do special education teachers think about the role of mathematical communication in the classroom?

3. METHODOLOGY

3.1. Research Design

The qualitative research approach is used to gain an in-depth understanding of the meanings attributed to people's beliefs, attitudes, and actions in a realistic and holistic manner (Merriam & Tisdell, 2016). Moreover, a phenomenological approach is a preferred qualitative strategy when the focus is to uncover, interpret, and explain how individuals experience a phenomenon and how they perceive it (Moustakas, 1994). Therefore, the current research was designed as a phenomenological study to examine special education teachers' thoughts on mathematical communication.

3.2. Participants

This study employed the convenience sampling technique of qualitative sampling, also known as (or purposeful) sampling, which allows for deeper inquiry by identifying information-rich scenarios that serve the purpose of the research (Patton, 2015). Ten special education teachers (7 females and 3 males) from various school levels [i.e., elementary school (n=2), middle school (n=5), and high school (n=3)] participated in the study on a voluntary basis. Before we obtained written informed consent from the participants, they were informed about all aspects of the study. In Turkey, 12 years of compulsory education consists of primary and secondary education. Compulsory primary education applies to children between the ages of 6 and 14. Primary education consists of four years of elementary school (grades 1, 2, 3, and 4) and four years of middle school (grades 5, 6, 7, and 8), with a choice of different programs. Secondary education, on the other hand, includes all general, vocational, and technical high schools (grades 9, 10, 11, and 12), which provide at least four years of compulsory formal or non-formal education to primary school graduates. In addition, the institutions and methods that provide education to students with SEN in Turkey vary depending on the needs and requirements of the student. There are various options such as special education and rehabilitation centers, private schools, and integrated schools. All teachers who participated in this study work in integrated public schools. These schools provide inclusive education to students with SEN alongside other students. In these schools, students are supported by special education teachers, and special education programs are implemented according to their needs. Each participant was also assigned a pseudonym for the purpose of ensuring confidentiality. Participants had been teaching from two years to sixteen years (M = 5.50, SD = 4.67). Moreover, all participants had bachelor's degrees in special education and received their teaching certificates from the Faculty of Education as part of their undergraduate studies. None of the participants earned their teaching credentials by completing additional pedagogy courses after graduation. An overview of the participants is presented in Table 1.

Table 1. *Demographic Information of Teachers Interviewed*

Pseudonym	Gender	Highest level of educational attainment	School level taught	Specialty area assigned	Overall teaching experience
Andrea	Female	Bachelor's degree	High school	Severe	7
Carla	Female	Bachelor's degree	Middle school	Severe	16
David	Male	Master's degree	Middle school	Severe	10
Eric	Male	Bachelor's degree	High school	Severe	5
Felix	Male	Bachelor's degree	Elementary school	Severe	7
Georgia	Female	Bachelor's degree	Middle school	Severe	2
Kyla	Female	Bachelor's degree	Middle school	Severe	2
Louisa	Female	Bachelor's degree	Elementary school	Severe	2
Roberta	Female	Bachelor's degree	Middle school	Severe	2
Simona	Female	Bachelor's degree	High school	Severe	2

3.3. Data Collection

Data collection was conducted through semi-structured online interviews via the "Zoom" videoconferencing platform, each lasting approximately 40 minutes. All interviews were scheduled at a time convenient for the participants. They were asked what they understood by mathematical communication in special education. Apart from the main study, a pilot study was also conducted with two volunteer special education teachers who were not involved in the actual study to determine if the interview questions were appropriate for the research purpose and to identify potential challenges that the researchers had not initially anticipated. The opinions of an expert in the field of special education for more than 10 years were also obtained. Based on the feedback from the pilot study and the expert opinions, some changes were made to the interview questions. For example, the question "What do you think a special education curriculum that emphasizes mathematical communication should look like in terms of general objectives, achievements, assessment and evaluation, learning areas, activities, etc.?" was removed from the interview questions because it was not fully within the scope of the research purpose. In addition, the question "How does mathematical communication affect the classroom environment?" was changed to "What do you think a mathematics class that focuses on mathematical communication should look like?" because it was not fully understood by the teachers in the pilot study. The final version of the questions in the interview guide was as shown in Table 2.

Table 2. *List of Semi-Structured Interview Questions*

Questions
• What do you think the concept of mathematical communication is?
• How important do you think mathematical communication is in special education?
• What indicators might reflect mathematical communication skills in special education?
• What do you think a mathematics class that focuses on mathematical communication should look like?

3.4. Data Analysis

Each interview was transcribed verbatim and analyzed thematically to identify, analyze, and report common patterns. NVivo (QSR International, 2012), a software program used for qualitative analysis, allowed the researchers to organize the data by coding them to label common themes. After the initial coding of all transcripts, the video recording of each interview was reviewed several times to verify the codes extracted from the transcripts. This process helped ensure the accuracy and internal consistency of the coding system by merging similar codes or removing unnecessary codes assigned to passages that did not exactly reflect the purpose of the study (Creswell & Poth, 2018). For example, various codes such as proficiency in processing mathematical symbols, synthesizing mathematical ideas, and expressing mathematical concepts considering relationships between them were combined to represent the broader theme of "*communicating mathematical ideas while considering cognitive competency*". Similarly, the theme of "*communicating mathematical ideas through embodiment*" was generated by combining the codes of embodying mathematics using it in everyday life, embodying mathematics using games, and embodying mathematics using body language. Moreover, participants were asked to review the transcribed interview data as part of the member-checking process to identify and avoid possible bias. All participants provided feedback and did not express any disagreement regarding the responses and comments given to them. In addition, two researchers, one of whom is an expert in special education and the other in mathematics education, with extensive experience in analyzing qualitative data were asked to serve as external raters. All codes were double-checked and revised until full agreement was reached on code definitions to ensure inter-coder reliability of the data (Miles et al., 2019).

3.5.Ethics Committee Approval

Approval for this study was granted by the Human Research Ethics Committee of Zonguldak Bülent Ecevit University on 21/09/2022 under registration number 215451.

4. FINDINGS

A cross-case analysis of responses from ten teachers revealed six themes for the role of mathematical communication in special education: (i) Communicating mathematical ideas through embodiment, (ii) Communicating mathematical ideas while considering cognitive competency, (iii) Listening, speaking, reading, or writing mathematical language, (iv) Developing positive attitudes towards using mathematical language, (v) Communicating mathematical ideas through repetitions, and (vi) Using mathematical language in other disciplines. Figure 1 summarizes findings on a single chart using the frequencies of emergent themes to visualize the data set. The numbers in parentheses next to each theme represent the number of teachers who mentioned it. The following sections present all themes in detail.

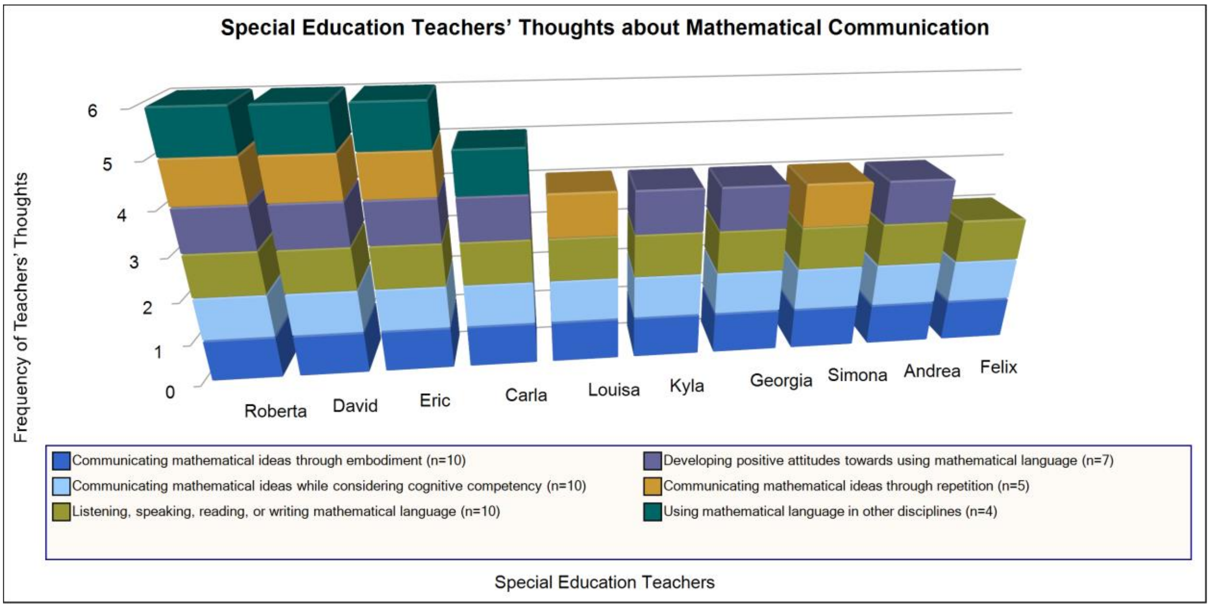

Figure 1. *Graphical Representation of Special Education Teachers' Thoughts about Mathematical Communication*

4.1. Communicating Mathematical Ideas through Embodiment

All participating teachers (*n*=10) emphasized that given the abstract nature of mathematics and its cognitive demands, it is quite difficult to directly teach mathematical concepts to students with SEN. They stated that it is always necessary to use teaching materials or concrete models from everyday life so that students can visualize and construct abstract mathematical concepts in their minds. In this regard, the teachers supported the view that mathematical communication is the embodiment of mathematics in teaching mathematical concepts and skills using concrete materials, real objects, or examples from students' daily lives. They also focused on the role of nonverbal communication such as pointing, gestures, nodding, facial expressions, and eye contact in learning and claimed that embodying mathematical ideas using body language facilitates mathematical communication. For instance, one of the teachers expressed that:

> …the most common example I encounter with children is the use of their fingers. When I tell them to show the number 4, they always use their fingers. This is the easiest way for them to embody the abstract concept and communicate it to us (Roberta).

And she put forward:

> … I think that mathematical ideas, concepts, and rules should be embodied especially for students who need special education because they have difficulty understanding abstract concepts. Therefore, they should be offered opportunities to work with tangible objects that they can manipulate at will. This lays the foundation for mathematical communication (Roberta).

In like manner, another teacher drew attention to the importance of embodying mathematics with real-life examples while communicating mathematics and continued as follows:

> …when it comes to mathematical communication, I think of integrating mathematical skills into everyday routines. Since mathematical concepts, relationships, and operations remain abstract and complex to my students, I try to open the door to the world of abstract thinking for them by making it concrete and meaningful through their daily lives or real-life examples (Eric).

In addition, some of the teachers (n=4) also remarked that the games used in the implementation of embodied learning helped to improve the quality of mathematical communication. They added that since games are fun, they are a way to develop the skills needed for effective communication by ensuring individuals' interactive participation in the learning process. As one of the teachers commented:

> …playing games is an important mechanism for helping students develop their mathematical communication skills as they interactively explain and justify their reasoning in class. I always try to encourage them to have an embodied learning experience so they can internalize abstract concepts through gestures or body movements while acting naturally in the games (Louisa).

4.2. Communicating Mathematical Ideas While Considering Cognitive Competency

All of the teachers (n=10) underlined the importance of the cognitive competence of students with special needs to be able to communicate mathematically. According to them, mathematical communication involves various cognitive processes such as attention, memory, perception, judgment, organization, comprehension, problem-solving, reasoning, and thinking. They explained that a developmental cognitive delay is a major barrier to learning and that a person with a cognitive disorder may have difficulty focusing on a conversation, maintaining a topic, remembering information, responding correctly, interpreting knowledge, or following directions. Therefore, the teachers agreed that mathematical communication in special education is an exchange of mathematical ideas that students perform depending on their cognitive abilities. For example, one of the teachers noted:

> …mathematics is a learning area that includes many cognitive processes such as planning, remembering facts or procedures, organizing ideas, finding solutions, and evaluating information. Since special students may have the skills required for these processes at different cognitive levels, mathematical communication is the ability to demonstrate mathematical thinking according to cognitive competency (David).

Hence, as another teacher expressed:

> …we conduct a needs assessment to find out exactly what our students need and what special support they need. In this way, we can more or less understand what each student can do cognitively in mathematics. I think that mathematical communication for these special students is the exchange and clarification of mathematical ideas based on their cognitive abilities (Simona).

4.3. Listening, Speaking, Reading, or Writing Mathematical Language

All those participating (*n*=10) pointed out that learning mathematics requires using mathematical language as effectively as possible to communicate ideas to others. They argued that mathematical communication is a process consisting of four basic mathematical language skills: listening, speaking, reading, and writing. They noted that all of these skills are linguistic prerequisites for developing mathematical language proficiency, but that students with SEN cannot be expected to fully master all of them. They believed that the key to success in mathematical communication for learners with special needs is to incorporate appropriate accommodations or modifications into classroom instruction and activities. For this reason, allowing students with writing difficulties to give their answers orally or making changes to the learning content to suit their abilities is an important support for effective interpersonal mathematical communication. For instance, one of the participants said:

> ...mathematical communication is the ability to listen, speak, read, or write the mathematical language, depending on the learner's needs...the fact that students with special needs attempt to express their mathematical statements and explanations to others orally or, if they are able, in writing is an indication that they are engaging in mathematical communication (Georgia).

Similarly, another participant maintained that:

> Speaking, reading, writing, or understanding mathematical language are each important means of communicating mathematical ideas to others. If my student can verbalize mathematical ideas, I listen to him. If he can understand or read the text but has difficulty speaking, I want him to express himself in different ways such as by writing. If he also has difficulty reading or following written directions, I facilitate his instruction. I adjust my instruction based on how easily my student can respond (Andrea).

4.4. Developing Positive Attitudes towards Using Mathematical Language

A common argument presented by most of the teachers (*n*=7) is that fostering the belief that everyone can do mathematics is influential in promoting mathematical communication in special education. In that sense, they insisted that building positive attitudes such as self-confidence, enjoyment, curiosity, interest, and motivation is essential for using mathematical language as much as possible to initiate mathematical communication. For example, one of the teachers stated:

> …they are students who need special education, and we try to teach them math, but I would say that sometimes they are quite hesitant or have a negative attitude toward the lessons. They have the impression that "I cannot do it." They find it difficult to understand the mathematical language that we use in class. So I think it is important to foster a positive attitude toward understanding and using mathematical language as a means of communicating ideas (Kyla).

Teachers added that reinforcement can have a positive effect on students' attitudes and help them show enthusiasm for using mathematical language. In this way,

> …the students' motivation to learn mathematics increases and I can better communicate with them mathematically (Carla).

4.5. Communicating Mathematical Ideas through Repetitions

Half of the teachers (*n*=5) mentioned that repetition has an important place in reinforcing skill acquisition for students with special learning needs. They recalled that the more often students are confronted with the same learning content, the more often they repeat it, the more likely they retain things in mind, and the more intense the transfer of mathematical ideas. They accepted the fact that repetition of new information improves the quality of mathematical communication because it provides mastery of new knowledge and skills through systematic practice. As one of the teachers simply put it:

> …each of my students is different, and they have difficulty transferring information into their long-term memory. They all need frequent repetition while working at different levels and at their own pace toward the goals they are trying to achieve. So what they all have in common is that they have certain routines that require repetition of prior knowledge and skills in each lesson. Therefore, repetition plays a vital role in mathematical communication, regardless of what level each individual is at or what their learning pace is (David).

4.6. Using Mathematical Language in Other Disciplines

Some of the teachers (*n*=4) argued that students with SEN learn in different ways and need various opportunities to talk about and share their mathematical thinking with others. Teachers explained that mathematics is highly intertwined with other school subjects and that using mathematical language in these subjects could create such opportunities. Therefore, they stated that mathematical communication is a process through which mathematical ideas can be consciously or unconsciously conveyed not only in mathematics classes but also in other school subjects where students feel more comfortable and confident in using mathematical language. The following excerpt from one of the teachers illustrates this point:

> …we can see many applications of mathematics in different scientific fields and disciplines. It also plays an important role throughout the school curriculum. Using mathematical language in other school subjects provides an effective way to communicate mathematically. For example, in my last PE class, one of my students was playing with a swing and suddenly started counting. I did not know she could count until then (Roberta).

5. RESULTS AND DISCUSSION

The results of this study demonstrate that special education teachers have a range of, but mutually supportive and compatible, thoughts about mathematical communication. Teachers emphasized that teaching mathematics to students with special needs is especially challenging due to some difficulties in communicating with them. They recognized that it takes extra time and effort for students to show their full potential because the way they communicate mathematically can be particularly different. Therefore, teachers discussed the concept of mathematical communication in special education from various perspectives. All those involved in this study proposed that communication in mathematics is characterized by several factors including the embodiment of mathematical knowledge, cognitive competencies of students, and effective use of mathematical language. Additionally, many teachers also noted that mathematical communication consists of some specific features such as developing positive attitudes toward learning mathematics, repeating mathematical facts, and using mathematics in multiple disciplines.

Believing that mathematical communication is the exchange of mathematical ideas through embodiment, all of the teachers in this study suggested that concrete models, actual objects, gestures, and body-based forms of participation help to make abstract concepts and their interrelations meaningful and relevant. Some of them added that incorporating games into the learning process encourages students to relate and communicate mathematical concepts with their concrete references. This is consistent with the idea that game-based environments can promote embodied learning of mathematics in an effort to convey mathematical ideas (Abrahamson et al., 2020). Teachers stressed that responding to students’ nonverbal communication patterns contributes to the development of their mathematical intuition and the effective translation of mathematical thinking into their actions. Therefore, unpacking the abstract world of mathematics and making it visible and tangible is seen as an essential practice to establish mathematical communication with students with SEN. In parallel, researchers support that in the embodied learning approach, the body and mind play an active role in the overall learning practice, and learning can be much more effective when the learner is placed at the center of the learning process through physical engagement with the learning context (Alibali, & Nathan, 2012; Chandler & Tricot, 2015; Kosmas et al., 2019). Therefore, embodied mathematics learning in special education can be said to have the potential to positively impact mathematical communication by providing opportunities to interact with the learning materials through hands-on activities.

Another point on which all teachers agree is the important role that the cognitive abilities of students with SEN play in demonstrating their mathematical communication skills. Teachers noted that while learning, remembering, and applying mathematical knowledge, students should go through certain cognitive processes such as analytical thinking, abstract reasoning, decision-making, and problem-solving that directly promote mathematical communication. Thus, doing mathematics requires engaging in communicative activities as students are encouraged to elaborate, clarify, and reorganize their mathematical ideas. Researchers refer to communication as an integral part of doing mathematics, which proceeds in the form of classroom discussions in which arguments are presented and defended (Hiebert, 1992; Schoenfeld, 1987). This is a highly social process as it involves arguing assumptions, making claims about established mathematical relationships, and developing ideas about whether the claims make sense (Lampert, 1991). In this way, students can construct knowledge and understanding through collaboration that they could not develop on their own (Schoenfeld, 1989). Hence, the cognitive competencies of students with SEN were considered important by teachers in order for individuals to gain knowledge and skills and to be able to communicate them for problem-solving.

All teachers in this study also expressed that a student's ability to communicate mathematically depends on the mastery of the four language skills in mathematics: listening, speaking, reading, and writing. They argued that using these language skills with confidence allows a person to understand and produce mathematical language for accurate and effective interpersonal communication in mathematics. In other words, to communicate with others in the class, the student expresses his or her mathematical thoughts by speaking or writing and in turn receives their thoughts by listening or reading. Speaking, listening, reading, and writing mathematical language are all integrated forms of receptive and productive language skills in communication that should occur regularly in mathematics classrooms (Thompson & Chappell, 2007). Moreover, teachers added that many students with SEN require some sort of accommodations or modifications based on their individual needs in using mathematical language to be successful in mathematical communication. Therefore, students who have difficulty perceiving or understanding visual, oral, or auditory presentations of instructional content can benefit from appropriate accommodations or, if necessary, modifications to fully access and make use of the presented content. Accommodations not only enable students to demonstrate their knowledge or skills in different modes but also provide them with alternative ways to access learning content (Ketterlin-Geller & Jamgochian, 2011). For cognitively overloaded students, on the other hand, instructional modifications can increase the accessibility of instruction by reducing the demands on their cognitive processing so that they can focus on learning basic knowledge and skills (Elliott et al., 2009).

In addition, many teachers pointed out that students with special needs often lack the motivation to participate in educational activities and many of them find it difficult to present their mathematical ideas. Since they show confusion and lack confidence in using mathematical language, they often do not dare to speak up or ask their teachers questions. It was underlined that building effective mathematical communication depends on promoting positive attitudes toward the use of mathematical language. Therefore, it is important to create opportunities in the classroom to foster enthusiasm for talking about mathematics (Taylor, 1991). In learning environments where the vast majority of students develop positive attitudes toward learning mathematics, such as the particular arrangement of groups, the accommodating pace of learning, the supportive role of the teacher, and the general conditions and atmosphere of the classroom, students frequently ask questions and provide explanations to their classmates (Leikin & Zaslavsky, 1997).

Also worth mentioning is repeated practice, which some teachers considered an important step in initiating mathematical communication for students with SEN. Teachers claimed that the more students practice, the more familiar and competent they become with their newly acquired knowledge and skills, and the more students internalize what they learn, the more doors are opened for them to explain, interpret, and apply mathematical concepts. In this context, Dahlin and Watkins (2000) argue that repetition has a profound effect on the mind and improves memorization, but can also be used to deepen and enhance understanding. Therefore, using routine practice as a teaching method can help students communicate mathematical facts, operations, or relations precisely and fluently. In other words, because learning through repetition promotes retention, students' long-term memory helps them somehow recall concepts, procedures, or ideas that will be needed in the future (Li, 2000).

Considerable emphasis was also placed on the fact that mathematics and other disciplines are in some way interrelated, and that encouraging students with SEN to use mathematical language by leveraging mathematics' connections to other school subjects of interest is a promising avenue to communicate mathematically. Therefore, one way to help students acquire mathematical communication skills is to integrate mathematics into other areas of knowledge whenever possible. This is because contexts that incorporate other fields can support the transfer of mathematics by providing practice or application of newly learned mathematical concepts or skills, illustrating the need to study mathematics (Frykholm & Meyer, 2002), and increasing students' interest and motivation to learn (Michelsen & Sriraman, 2009).

6. RECOMMENDATIONS

Overall, the results suggest that mathematical communication is an essential component of mathematics teaching and learning in special education. Given the abstract nature of mathematics, mathematical communication is seen as a way to convey mathematical ideas through embodiment. It can also be seen as a process of rationalizing and justifying ideas that become embodied objects of inquiry and interpretation by using mathematical language through listening, speaking, reading, or writing according to cognitive competencies. Therefore, developing a positive attitude toward the use of mathematical language is believed to help students with SEN to think about new learning experiences and to facilitate communication of the results of their thinking with others, not only in mathematics classrooms but also in other school subjects in which they feel more comfortable. Also noteworthy is the claim that the repetition of new information can enhance mathematical communication by enabling the acquisition of new knowledge and skills through systematic practice. The emergence of such diverse thinking may shed more light on understanding mathematical communication from a pedagogical perspective. This research can also be extended to investigate the consistency between what special education teachers think about mathematical communication and their teaching practices. Another important issue that needs to be investigated is whether what they think about mathematical communication influences their students' perceptions of mathematical communication. Last but not least, the present study was limited in its potential to explore the thoughts of special education teachers responsible for teaching students with severe disabilities. Therefore, it is worthwhile to conduct further research with teachers from other disability areas of specialization.

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9. References

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., et al. (2020). The future of embodied design for mathematics teaching and learning. *Frontiers in Education*, *5*(147), 1-29. https://doi.org/10.3389/feduc.2020.00147

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences*, *21*(2), 247-286. http://doi.org/10.1080/10508406.2011.611446

Barwell, R. (2008). Discourse, mathematics and mathematics education. In N. H. Hornberger (Ed.), *Encyclopedia of Language and Education* (pp. 317-328). Springer.

Chandler, P., & Tricot, A. (2015). Mind your body: The essential role of body movements in children's learning. *Educational Psychology Review, 27*(3), 365-370. https://doi.org/10.1007/s10648-015-9333-3

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). Classroom discussions using math talk in elementary classrooms. *Math Solutions, 11*, 1-3.

Chard, D. (2003). *Vocabulary strategies for the mathematics classroom*. Houghton Mifflin.

Chval, K. B., & Khisty, L. L. (2009). Bilingual Latino students, writing and mathematics: A case study of successful teaching and learning. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 128-144). Multilingual Matters.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education*, *28*(3), 258-277. https://doi.org/10.5951/jresematheduc.28.3.0258

Cobb, P., Wood, T., & Yackel, E. (1994). Discourse, mathematical thinking and classroom practice. In E. A. Forman, N. Minick & C. Addison Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-120). Oxford University Press.

Cooke, B. D., & Buchholz, D. (2005). Mathematical communication in the classroom: A teacher makes a difference. *Early Childhood Education Journal*, *32*(6), 365-369. https://doi.org/10.1007/s10643-005-0007-5

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches*. (4th ed.). SAGE Publications.

Dahlin, B., & Watkins, D. (2000). The role of repetition in the processes of memorising and understanding: A comparison of the views of German and Chinese secondary school students in Hong Kong. *British Journal of Educational Psychology*, *70*(1), 65-84. https://doi.org/10.1348/000709900157976

Elliott, S. N., Kurz, A., Beddow, P., & Frey, J. (2009). Cognitive load theory: Instruction-based research with applications for designing tests. *Paper presented at the Annual Convention of the National Association of School Psychologists* (Vol. 24, pp. 1-22).

Ernest, P (1988). The impact of beliefs on the teaching of mathematics. *Paper prepared for ICME VI*. http://www.ex.ac.uk/~PErnest/impact.htm

Ernst-Slavit, G., & Slavit, D. (2007). Educational reform, mathematics, & diverse learners: Meeting the needs of all students. *Multicultural Education*, *14*(4), 20-27.

Frykholm, J. A., & Meyer, M. (2002). Integrated instruction: Is it science? Is it mathematics?. *Mathematics Teaching in the Middle School*, *7*(9), 502-508. https://doi.org/10.5951/MTMS.7.9.0502

Fuchs, L. S., Malone, A. S., Schumacher, R. F., Namkung, J., & Wang, A. (2017). Fraction intervention for students with mathematics difficulties: Lessons learned from five randomized controlled trials. *Journal of Learning Disabilities*, *50*(6), 631-639. https://doi.org/10.1177/0022219416677249

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*(3), 1202-1242. https://doi.org/10.3102/0034654309334431

Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture on the postmodern age*. Teachers College Press.

Hiebert, J. (1988). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics*, *19*(3), 333-355. http://www.jstor.org/stable/3482522

Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal, 26*(2), 160-189. https://doi.org/10.2307/1163030

Hoyles, C. (1985). What is the point of group discussion in mathematics?. *Educational Studies in Mathematics*, *16*(2), 205-214. https://doi.org/10.1007/PL00020740

Jackson, H., & Neel, R. (2006). Observing mathematics: Do students with EBD have access to standards-based mathematics instruction? *Education and Treatment of Children*, *29*(4), 593-614. https://www.jstor.org/stable/42900555

Jung, H. Y., & Reifel, S. (2011). Promoting children's communication: A kindergarten teacher's conception and practice of effective mathematics instruction. *Journal of Research in Childhood Education*, *25*(2), 194-210. https://doi.org/10.1080/02568543.2011.555496

Kaya, D., & Aydın, H. (2016). Elementary mathematics teachers' perceptions and lived experiences on mathematical communication. *Eurasia Journal of Mathematics*, *Science and Technology Education*, *12*(6), 1619-1629. https://doi.org/10.12973/eurasia.2014.1203a

Ketterlin-Geller, L. R., & Jamgochian, E. M. (2011). Instructional adaptations: Accommodations and modifications that support accessible instruction. In S. N. Elliott, R. J. Kettler, P. A. Beddow, & A. Kurz (Eds.), *Handbook of accessible achievement tests for all students: Bridging the gaps between research, practice, and policy* (pp. 131-146). Springer. https://doi.org/10.1007/978-1-4419-9356-4_7

Kinman, R. L. (2010). Communication speaks. *Teaching Children Mathematics*, *17*(1), 22-30. https://doi.org/10.5951/TCM.17.1.0022

Kosmas, P., Ioannou, A., & Zaphiris, P. (2019). Implementing embodied learning in the classroom: effects on children's memory and language skills. *Educational Media International*, *56*(1), 59-74. https://doi.org/10.1080/09523987.2018.1547948

Kostos, K., & Shin, E. K. (2010). Using math journals to enhance second graders' communication of mathematical thinking. *Early Childhood Education Journal*, *38*(3), 223-231. https://doi.org/10.1007/s10643-010-0390-4

Lambert, R., Sugita, T., Yeh, C., Hunt, J. H., & Brophy, S. (2020). Documenting increased participation of a student with autism in the standards for mathematical practice. *Journal of Educational Psychology*, *112*(3), 494-513. https://doi.org/10.1037/edu0000425

Lampert, M. (1991). Connecting mathematical teaching and learning. In E. Fennema, T. P. Carpenter, & S. Lamon (Eds.), *Integrating research on teaching and learning math*ematics (pp. 121-152). State University of New York Press.

Lei, Q., Xin, Y. P., Morita-Mullaney, T., & Tzur, R. (2020). Instructional scaffolds in mathematics instruction for English learners with learning disabilities: An exploratory case study. *Learning Disabilities: A Contemporary Journal*, *18*(1), 123-144.

Leikin, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education*, *28*(3), 331-354. https://doi.org/10.5951/jresematheduc.28.3.0331

Li, S. (2006). Practice makes perfect: A key belief in China. In F. K. S. Leung, K. D. Graf, & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West* (pp. 129-138). Springer. https://doi.org/10.1007/0-387-29723-5_8

Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *22*(1), 58-74.

Martinez, J. G. R. (2001). Thinking and writing mathematically: Achilles and the tortoise as an algebraic word problem. *Mathematics Teacher*, *94*(4), 248-252. https://doi.org/10.5951/MT.94.4.0248

Matteson, S. (2006). Mathematical literacy and standardized mathematical assessments. *Reading Psychology*, *27*(2-3), 205-233. https://doi.org/10.1080/02702710600642491

McLennan, D. P. (2014). Making math meaningful for young children. *Teaching Young Children*, *8*(1), 20-22.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation* (4th ed.). Jossey-Bass.

Michelsen, C., & Sriraman, B. (2009). Does interdisciplinary instruction raise students' interest in mathematics and the subjects of the natural sciences? *ZDM-Mathematics Education*, *41*, 231-244. https://doi.org/10.1007/s11858-008-0161-5

Miles, M. B., Huberman, A. M., & Saldana, J. (2019). *Qualitative data analysis: A methods sourcebook* (4th edition). SAGE Publications.

Morgan, C. (2016). Communicating mathematically. In S. Johnston-Wilder, P. Johnston-Wilder, C. Lee & D. Pimm (Eds.), *Learning to teach mathematics in the secondary school: A companion to school experience* (pp. 148-163). Routledge.

Moustakas, C. (1994). *Phenomenological research methods*. SAGE Publications. https://dx.doi.org/10.4135/9781412995658

National Council of Teachers of Mathematics. (NCTM). (2000). *Principles and standards for school mathematics*. NCTM.

National Council of Teachers of Mathematics. (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

National Council of Teachers of Mathematics. (NCTM). (2020). *Standards for the preparation of middle level mathematics teachers*. NCTM.

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

O'Halloran, K. L. (2000). Classroom discourse in mathematics: A multisemiotic analysis. *Linguistics and Education*, *10*(3), 359-388. https://doi.org/10.1016/S0898-5898(99)00013-3

Ontario Ministry of Education (2005). *Education for all. The report of the expert panel on literacy and numeracy instruction for students with special education needs, kindergarten to grade 6*. Queen's Printer for Ontario.

Ontario Ministry of Education (2006). *A guide to effective instruction in mathematics, Kindergarten to grade 6: A resource in five volumes from the Ministry of Education.* Queen's Printer for Ontario.

Ontario Ministry of Education (2020). *The Ontario curriculum, grades 1-8: Mathematics*. Queen's Printer for Ontario.

Organisation for Economic Co-operation and Development. (OECD). (2013). *PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy*. OECD Publishing.

Owens, B. (2006). The language of mathematics: *Mathematical terminology simplified for classroom use*. (Master's thesis). East Tennessee State University.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332. https://doi.org/10.3102/00346543062003307

Pape, S. J., Bell, C. V., & Yetkin, İ. E. (2003). Developing mathematical thinking and self-regulated learning: A teaching experiment in a seventh-grade mathematics classroom. *Educational Studies in Mathematics*, *53*(3), 179-202. https://doi.org/10.1023/A:1026062121857

Patchan, M. M., Rambo-Hernandez, K. E., Deitz, B. N., & McNeill, J. (2022). Using peer assessment to improve middle school mathematical communication. *The Journal of Educational Research*, *115*(2), 146-160. https://doi.org/10.1080/00220671.2022.2074948

Patton, M. Q. (2015). *Qualitative research & evaluation methods: Integrating theory and practice*. (4th ed.). SAGE Publications.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Information Age.

Pourdavood, R. G., & Wachira, P. (2015). Importance of mathematical communication and discourse in secondary classrooms. *Global Journal of Science Frontier Research: Mathematics and Decision Sciences*, *15*(10), 9-20.

Purcell Gates, V., Melzi, G., Najafi, B., & Orellana, M. F. (2011). Building literacy instruction from children's sociocultural worlds. *Child Development Perspectives*, *5*(1), 22-27. https://doi.org/10.1111/j.1750-8606.2010.00144.x

Q. S. R. International (2012). *NVivo qualitative data analysis software (Version 10) [Computer software]*. QSR International Pty Ltd.

Ramirez, N., & Celedón-Pattichis, S. (2012). Second language development and implications for the mathematics classroom. In S. Celedón-Pattichis & N. Ramirez (Eds.), *Beyond good teaching: Advancing mathematics education for ELLs* (pp. 19-37). NCTM.

Rubenstein, R. N., & Thompson, D. R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics*, *9*(2), 107-112. https://doi.org/10.5951/TCM.9.2.0107

Santos, M. I., Breda, A., & Almeida, A. M. (2015). Brief report: Preliminary proposal of a conceptual model of a digital environment for developing mathematical reasoning in students with autism spectrum disorders. *Journal of Autism and Developmental Disorders*, *45*(8), 2633-2640. https://doi.org/10.1007/s10803-015-2414-9

Schmidt, W. H. (2004). A vision for mathematics. *Educational Leadership*, *61*(5), 6-11.

Schoenfeld, A. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1989). Ideas in the air: Speculations on small group learning, environmental and cultural influences on cognition and epistemology. *International Journal of Educational Research*, *13*(1), 71-88. https://doi.org/10.1016/0883-0355(89)90017-7

Shellard, E. G. (2004). Helping students struggling with math. *Principal*, *84*(2), 40-43.

Smith, S. S. (2012). *Early childhood mathematics* (5th ed.). Pearson Education, Inc.

Staples, M. E., & Truxaw, M. P. (2010). Enhancing language, enhancing learning: Augmenting mathematics teachers' capacity in their linguistically diverse classrooms. In Brosnan, P., Erchick, D. B., & Flevares, L. (Eds.), *Proceedings of the 32th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Educatio*n (pp. 1337-1345). The Ohio State University.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2015). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. In L. B. Resnick, C. S. C. Asterhan, & S. N. Clark (Eds.), *Socializing intelligence through academic talk and dialogue* (pp. 375-388). American Educational Research Association. https://doi.org/10.3102/978-0-935302-43-1_29

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership*, *61*(4), 12-17.

Taylor, L. (1991). Facilitating effective mathematical communication. *Research and Teaching in Developmental Education*, *7*(2), 93-99.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 127-146). Macmillan.

Thompson, D. R., & Chappell, M. F. (2007). Communication and representation as elements in mathematical literacy. *Reading & Writing Quarterly*, *23*(2), 179-196. https://doi.org/10.1080/10573560601158495

Turner, R., Blum, W., & Niss, M. (2015). Using competencies to explain mathematical item demand: A Work in progress. In K. Stacey, & R. Turner (Eds.), *Assessing mathematical literacy: The PISA experience* (pp. 85-116). Springer. https://doi.org/10.1007/978-3-319-10121-7_4

Ugalde, L., Santiago-Garabieta, M., Villarejo-Carballido, B., & Puigvert, L. (2021). Impact of interactive learning environments on learning and cognitive development of children with special educational needs: A literature review. *Frontiers in Psychology*, *12*, 674033. https://doi.org/10.3389/fpsyg.2021.674033

Whitin, P., & Whitin, D. J. (2003). Developing mathematical understanding along the yellow brick road. *Young Children*, *58*(1), 36-40.