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Types of Visual Representations Used in Problem Solving and Its Relationship with Problem Solving Performance[1]

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Abstract: The purpose of the study was to examine the types of representations that secondary school students used while solving problems and the relationship between these representation types and students' problem-solving performances. The research was carried out in the Western Black Sea Region in Turkey in the spring term of the 2020-2021 academic year. The study group of the research consisted of 212 students (5th, 6th, 7th, and 8th grades) from each grade level of four secondary schools, selected by convenience sampling method. The Visualization Test consisted of 10 problems was used to collect the data of the research. The descriptive statistics frequency (f) and percentage (%) were used to determine the types of representations used by students while solving the problems and correlation analysis was conducted to determine the existence of the relationship between the scores the students got from the Visualization Test and the representation types they used. The results concluded that secondary school students mostly used visual representations (schematic and pictorial representations). Moreover, the results indicated that there was a moderate positive relationship between the Visualization Test total scores and total schematic scores, while there was a statistically high level of negative correlation between the Visualization Test total scores and total pictorial representation scores. Additionally, the results showed that there was a moderately negative relationship between the Visualization Test total scores and total computational scores.

Keywords: *Visual Representations, Schematic Representations, Pictorial Representations, Problem Solving.*

1. INTRODUCTION

An ordered and complicated transformation process that shapes and depicts a notion that occurs between mental pictures makes up the visualizing experience (Men et al., 2018). Mathematics is one subject where this visualization technique is successfully applied. Utilizing visual representations in place of abstract ideas is known as visualization in mathematics (Wheatley, 1998). This aspect of visualization drives the analytical transformation of existing data, making information more useful. With the help of useful methods such as graphs, diagrams, or different geometric shapes, abstract concepts are represented (Sağlam & Bülbül, 2012). In its broadest sense, the term "visualization" refers to

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the concretization process, which is the result of a high-level cognitive process for determining, explaining, and demonstrating the operation of objects, processes, or events using images, graphs, charts, diagrams, tables, and other visual aids (Sung et al., 2010). The definition of visualization by Zazkis et al. (1996, p. 441) is “Visualization is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses”. According to Zazkis et al. (1996), any mental construction of things or procedures that a person links with things or events they believe to be external might be considered a visualization act. An alternative to visualization is the development of images that the person associates with real-world items or processes on an external surface, such as paper, a whiteboard, or a computer screen. In other words, the concept of visualization is the act of representing a data based on the sense of sight (Yılmaz & Argün, 2013). In addition, while defining the concept of visualization, it is possible to state that what people gain through their senses has a strong connection with the concepts they have internalized (Işık & Konyalıoğlu, 2005). When the definitions of the concept of visualization in literature are examined, the common point of all these definitions is based on the principle of concretization and representation of the acquired knowledge (Tekin & Konyalıoğlu, 2020). In general, the visual thinking potential of people, which forms the basis of visualization, is the explanation of the differences in the tendency to visualize. In other words, the use and variety of visual representations differ from person to person. However, the common denominator of these differences is for the purpose of transforming, producing, manipulating, working, recreating, or representing the acquired knowledge into a meaningful visual, and the result of this action expresses the degree of mental processing ability of the person (Rha et al., 2010).

The learning process is quick and permanent when vocally communicated information is made concrete and visualized (Bozkurt, 2012). The reason why the use of visualization accelerates the learning process is that it makes it easier to understand the information, to reveal the meanings and complexity of the rules more easily. In a more concise statement, visualization is a pragmatic method of the process required for the internalization of knowledge. For instance, teaching a student about fractions in a mathematics lesson using visual representations of the operations carried out in the education is a factor that helps the student understand what the operations mean and better understand the subject rather than making it simpler for them to carry out operations (Baki, 2018). The advantages of this method are also highlighted in visualization theories. Dienes' (1960) "multiple embodiment principle" is one of the most striking and well-known theories. One of his multiple embodiment principles, which states that to abstract a mathematical structure successfully, one must meet it in a variety of diverse settings to comprehend its purely structural qualities, characterizes his use of abstraction (Dienes, 1960, as cited in Zhang et al., 2015). By giving students a variety of models, you can help them move closer to general abstraction. The abstraction will be more widely applicable the more broadly grounded it is. In other words, the likelihood that the final abstract idea will be relevant to a wide range of applications and situations increases if the abstraction is the result of grouping together the common qualities of many different situations (Dienes, 1967). In this context, visualization is the process of establishing a connection because of associating existing conditions, not reaching a solution, or finding a solution in a single attempt (Tekin & Konyalıoğlu, 2020). According to Alsina and Nelsen (2006), visualization can be used effectively not only when solving problems, but also in the field of education and training, to make a mathematical concept more concrete, to define concepts and learn their form, or to create a proof.

2. LITERATURE REVIEW

When the results of the visualization studies in the literature were examined, it was seen that the use of visualization in mathematics contributed to the problem-solving skills of the students (Anwar et al., 2019; Dockendorff & Solar, 2018; Ergan & Özsoy, 2021; Hegarty & Kozhevnikov, 1999; Ke & Clark, 2018; Kükey et al., 2019; Van Garderen & Montague, 2003; Zeybek & Saygı, 2018) and improved their computational skills (Bunar, 2011; Gökbulut et al., 2013). In addition, it was suggested that there was a positive relationship between visual reasoning skills and spatial visualization skills (Akkan et al., 2018). Additionally, classroom contexts and students' experiences with visual representations in the classroom can shape their preferences and affect how they use different representations to solve problems. A study that examined this issue was conducted by Stieff and Wilensky (2003). They investigated how students' experiences with visual representations in the classroom affected their problem-solving strategies. The results of the study revealed that students who were more familiar with dynamic, interactive visual representations were more likely to use them to solve problems in comparison to students who were more familiar with static, pictorial representations were more likely to use these representations. Correspondingly, studies emphasized the importance of providing students with a variety of visual representations to support problem-solving and highlighted the role of tasks and contexts in shaping students' use of representations (Hegedus & Tall, 2016). Moreover, depending on the findings obtained, it was seen that visualization skills were affected not only by the method used, but also by different demographic variables (i.e., education level of parents) (Delibaş, 2020) and various factors (i.e., abstract thinking skills in mathematics) (Başer & Uysal Koğ, 2011).

The experimental study of Hegarty and Kozhevnikov (1999), in which they investigated the relationship between students' use of visual expressions, their spatial abilities and solving mathematical problems, also provides remarkable data on visualization. In this study, students' visual uses were defined in two types of visual representation classes as "schematic representations" and "pictorial representations". Schematic representation was defined as drawings that represented spatial relationships between objects in the problem and were made by imagining spatial transformations. While pictorial representation was defined as depicting the image of items such as person-object presented in the problem in detail. As a result of the research, it was determined that there was a positive relationship between the use of schematic representation and problem-solving performance, and the use of pictorial representation had a negative relationship with problem solving performance. Similarly, Ergan and Özsoy (2021) examined the types of visual representations created by fourth-grade primary school students in the problem-solving process and how the types of visual representations were distributed based on solving the problem correctly. As a result of the research, three types of representation (schematic, pictorial and computational) created by the students during the problem-solving process were defined. Accordingly, computational representation was computing of the numerical values presented in the problem or the representation by converting them into equations. While it was seen that most of the students who created schematic representation in the problem-solving process gave the correct answer to the problem, it was seen that the majority of the students who were created pictorial representations gave the wrong answer to the problem. In addition, it was observed that more than half of the students utilizing computational representation gave the wrong answers to the problems. Similarly, Van Garderen and Montague (2003) also examined how students use visual images while solving mathematical problems. As a result of the research, it was determined that gifted students used more visual-spatial representations. They also concluded that there was a positive relationship between problem solving success and the use of schematic representations, and a negative relationship with the use of pictorial representations. Another study, which concluded that schematic representations were the most common type of representation used by secondary school students in the mathematical problem-solving process, belongs to Ke and Clark (2018). In this study, it was concluded that schematic representations were used more frequently than computational representations and had a positive relationship with mathematical problem-solving success. On the other hand, Anwar et al. (2019) focused on secondary school students' use of schematic representation in problem solving processes, and as a result of this study, it was determined that the students had an idea about the next action to be taken with the use of schematic representation, and this allowed students to better understand the problem. Examining the types of representations that secondary school students used while solving problems and its relationship with problem-solving performance was important for mathematics education literature because visual representations could provide a powerful tool for helping students understand mathematical concepts and solve problems (Hegedus & Tall, 2016). However, different types of representations might be effective for different tasks or learning goals. By examining the relationship between representation types and problem-solving performance, researchers and educators could identify which types of representations were most effective for supporting different types of problem-solving tasks. Additionally, understanding how students used representations while solving problems could provide insights into their thought processes and problem-solving strategies. This information could help teachers better understand their students' learning needs and provide more targeted instruction (Lesh & Zawojewski, 2007). In the light of all these research findings, it was thought that it was important to examine the visualization representations of secondary school students used while solving problems. In this context, answers to the following problems were sought:

1. What types of representations do secondary school students use while solving the problems in the Visualization Test?

2. Is there a significant relationship between the secondary school students' total scores get from the Visualization Test and the types of representation they use while solving the problems in the Visualization Test?

3. METHODOLOGY

3.1. Research Model

In this study, the correlational survey research design, one of the survey designs, was used. Creswell (2007) defined the correlation method as a quantitative research methodology where researchers conducted statistical correlation analysis processes to determine whether two or more variables are correlated and the strength of that correlation. The relationship was determined based on the students' total scores of Visualization Test and scores from each type of representation of the Visualization Test. It is important to note that, a correlational study cannot be associated with causality or cause-effect relationship (Korwar, 2021).

3.2. Sample

The research was carried out in a province in the Western Black Sea Region in Turkey in the spring term of the 2020-2021 academic year. The study group of the research consisted of 212 students (5th, 6th, 7th, and 8th grades) from each grade level of four secondary schools, two from village secondary schools and two from central secondary schools, selected by convenience sampling method. The use of this sampling method brought practicality to the researchers and accelerated the research process (Yıldırım & Şimşek, 2018). Of the participants, 100 (47.16%) were male and 112 (52.83%) were female students.

3.3. Data Collection Tools

The Visualization Test developed by Budram (2009) was used to collect the data of the research. This test consists of non-routine problems that can be solved using visual representations. First of all, the test was adapted and translated into Turkish. Then, the draft document was translated once more into English. The translations revealed a correspondence between the Turkish and English versions. Afterwards, the pilot study was started by taking the opinions of two mathematics educators and a Turkish language expert. A total of 40 students from 5th, 6th, 7th, and 8th grade students from a public secondary school participated in the pilot study. While the first version of the test used in the pilot study consisted of 11 problems, one of these problems was removed because no student gave an answer, and a problem was reviewed in order to be more understandable. After the pilot study, the final version of the test consisting of 10 problems was reached.

3.4. Analysis of Data

The data of the research were collected in two class hours, in the classroom environment, by providing equal conditions for students at all levels. The analysis of the data covers the process of revealing the findings in the collected data (Bogdan & Biklen, 2007). A statistical package program was used to analyze the data obtained in this study. First, students' solutions were divided into two categories, schematic and pictorial representations, according to the types of visual representations they used while solving the problems. However, when the student papers were examined, it was seen that there were solutions in which they did not use any visual elements, and computational representations were determined as the third category. Schematic representation was defined as drawings that represented spatial relationships between objects in the problem and were made by imagining spatial transformations, while pictorial representation was defined as depicting the image of items such as person-object presented in the problem in detail (Hegarty & Kozhevnikov, 1999). An example of the solution using schematic representation is given in Figure 2. In this solution, the student made her drawing using spatial relationships between people in the ticket queue given in the problem. However, in Figure 5, which is an example of the solution using pictorial representation, the student drew only the image of the person and the elevator presented in the problem by ignoring the relationship between the problem items and numerical values. Computational representation, on the other hand, was computing of the numerical values presented in the problem or the representation by converting them into equations without using visuals (Ergan & Özsoy, 2021). An example for the solution used computational representation is given Figure 7. In this solution, the student only used the numerical values in the problem and solved the problem by setting up an equation, instead of using drawings showing the relationship between the variables in the problem.

The descriptive statistics frequency (f) and percentage (%) were used to determine the types of representations used by students while solving the problems, which is the first research problem. In the analysis of the second research problem, correlation analysis was conducted to determine the existence of the relationship between the two variables (Karasar, 2005). To examine the relationships between the scores, the students got from the Visualization Test and the representation types they used, the Pearson Product Moments Correlation Coefficient was calculated between the total scores and the scores obtained from each representation type. While calculating the total scores, the number of correct answers given to the problems in the Visualization Test was taken into consideration. Moreover, while calculating the scores obtained from each representation type, it was taken into account how many problems that representation type was used in. Since the data should be normally distributed in order to calculate this coefficient, normality tests were performed for all four variables (total scores, schematic representation scores, pictorial representation scores, and computational representation scores) and it was observed that the data showed normal distributions (George & Mallery, 2010). The data obtained from the study were coded by two coders and calculated using the formula of Miles and Huberman (1994) Reliability = Consensus / (Agreement +

Disagreement) and the reliability was found to be 98%. For example, there was a difference of opinion in the solution of a seventh-grade student in Figure 1.

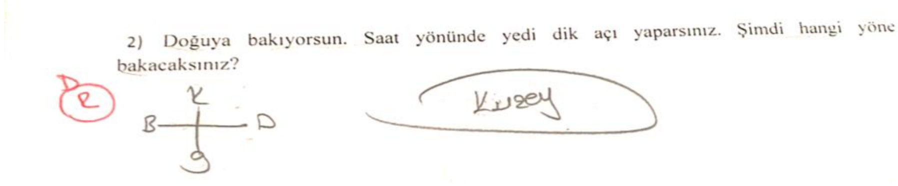

Figure 1. *A Seventh-Grade Student's Solution*

In the solution, the student drew the directions with the arrows and reached the correct answer directly. One of the researchers evaluated this solution in the category of pictorial representation. The other researcher, on the other hand, argued that this representation could be included in the schematic representation since it did not contain any pictorial detail and contains the place-direction relationship of the problem.

4. FINDINGS

In this section, the findings obtained as a result of the data analysis regarding the types of representations used by the secondary school students participating in the study while solving problems and the relationship between these representation types and the problem-solving performances of the students are included.

4.1. Findings Regarding the Types of Representation Students Use While Solving Problems

When the solutions of the students in the Visualization Test were examined, a total of 2108 solutions were obtained for 10 problems from each student, since 12 problems were left blank. The distribution of these solutions according to the types of representation is given in Table 1.

Table 1. *Distribution of Solutions According to Representation Types*

Representation Type					Total	
	Correct		Incorrect			
	f	%	*f*	%	*f*	%
Schematic	784	97.63	15	2.37	803	38.09
Pictorial	0	0	442	100	442	20.97
Computational	372	43.11	491	56.89	863	40.94

When Table 1 was examined, it was seen that secondary school students mostly benefited from schematic and pictorial representations (59.06%), which were visual representations. Looking at the representation types one by one, the most common representation type used by students was computational representation (40.94%), which was a non-visual representation type. In addition, the most frequently used visual representation type by students was schematic representation (38.09%), and the least used representation type was pictorial representation (20.97%). Considering whether the problems were solved correctly according to the representation type, it was seen that most of the solutions using schematic representation (97.63%) reached the correct answers. On the other hand, it was found that none of the secondary school students who tried to produce solutions to problems with the use of pictorial representation could reach the correct answer.

4.1.1. Findings Regarding the Schematic Representations Used by Students

Some examples of schematic representations used by students while solving problems are given below. The common feature of these representations is that they are drawings that represent spatial and mathematical relations between the objects in the problem and are made by imagining spatial transformations. For example, a seventh-grade student solved the fifth problem as in Figure 2.

Figure 2. *A Seventh-Grade Student's Solution Using Schematic Representation*

It was seen that the student, who answered the fifth problem with a schematic representation, drew the people in the ticket queue and reached the correct solution of the problem by positioning them according to the situation given in the problem. Another student's solution using schematic representation is given in Figure 3.

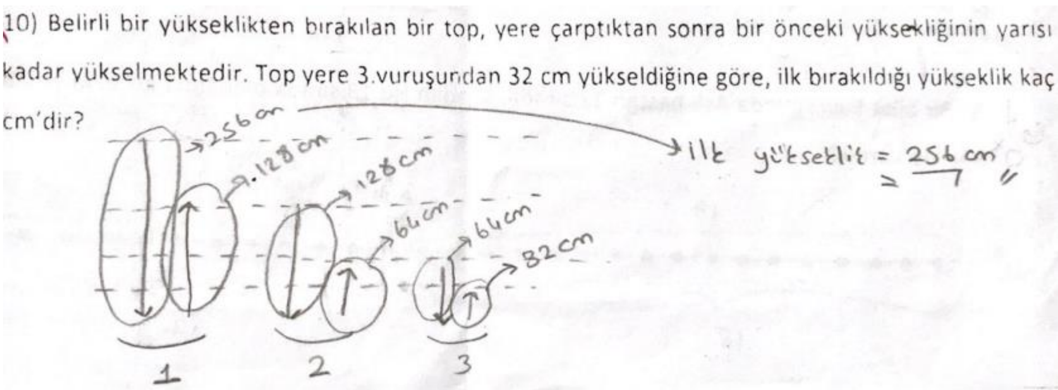

Figure 3. *An Eighth-Grade Student's Solution Using Schematic Representation*

When the solution of the student who answered the tenth problem with a schematic representation was examined, the student showed by drawing a figure that from the last state to the first state, it rose by half the previous height each time. Since he gave that he had risen 32 cm in the last case, he found the height at which he was left as 64 cm and the previous height as 128 cm, and he calculated the height at which he was left at the beginning as 256 cm. Most of the students using the schematic representation gave correct answers to the problems. But there were also students who used schematic representation and gave wrong answers. For example, an eighth-grade student's solution using schematic representation is given in Figure 4.

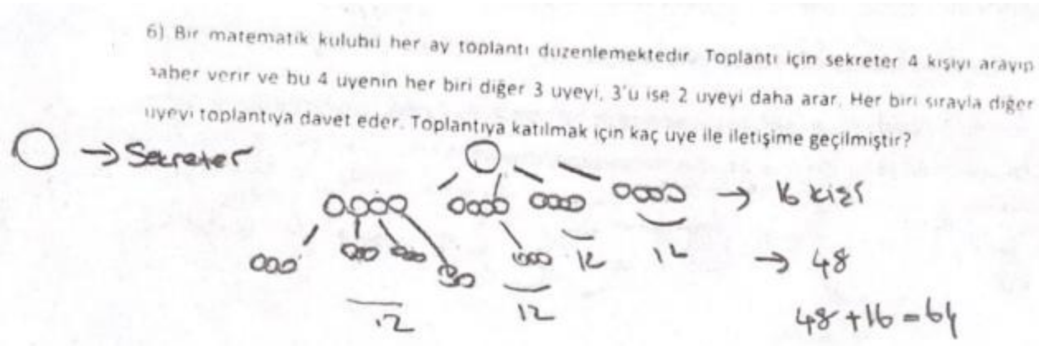

Figure 4. *An Eighth-Grade Student's Solution Using Schematic Representation*

When the solution of the student using schematic representation in the sixth problem was examined, the student drew circles to represent the secretary and the members. It was observed that the student did not deal with the operations sequentially in their solution, but repeated them on top of each other. It was seen that the schematic representation contributed to the student's solution, but the solution was wrong as no relationship could be established between the numerical operations and the figure.

4.1.2. Findings Regarding Pictorial Representations Used by Students

Some of the pictorial representations used by students while solving problems are given below. The common feature of these representations is that they contain detailed images of people or objects given in the text of the problem. For example, a fifth-grade student solved the third problem as in Figure 5.

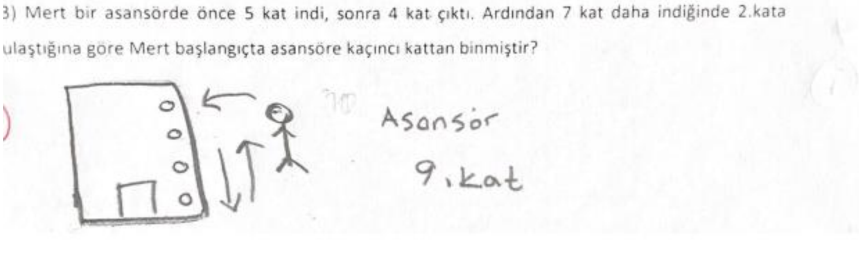

Figure 5. *A Fifth-Grade Student's Solution Using Pictorial Representation*

When the solution of the student who answered the third problem with pictorial representation was examined, it was seen that the student gave the wrong answer because she made a drawing for the image of the objects in the problem, which does not include the relationship between the problem items and numerical values. A sixth-grade student's solution using pictorial representation is given in Figure 6.

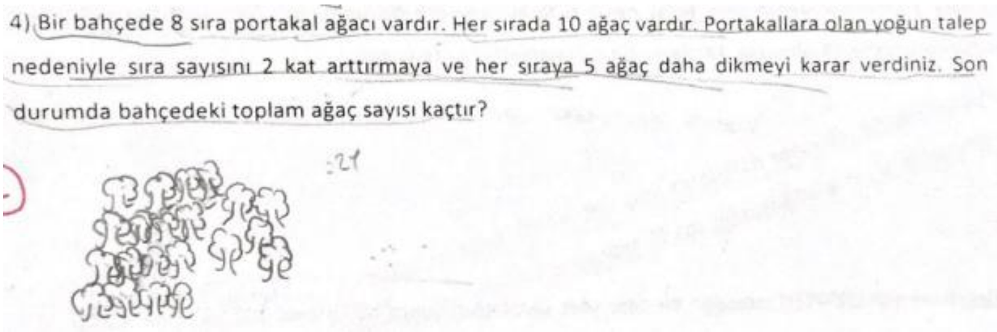

Figure 6. *A Sixth-Grade Student's Solution Using Pictorial Representation*

When the solution of the student who answered the fourth problem with pictorial representation was examined, it was seen that the student tried to solve the problem by ignoring the relationships between the items given in the problem and only drawing the objects on the paper, so he reached the wrong answer. When all solutions in this representation type were examined; it was determined that none of the secondary school students who tried to produce solutions to problems with the use of pictorial representation could reach the right answer.

4.1.3. Findings Regarding the Computational Representations Used by Students

Some examples of computational representations, which are non-visual representation types used by students while solving problems, are given below. The common feature of these representations is that students solve problems by converting the numerical values given in the problem into equations or using operations without using any visuals. For example, a seventh-grade student solved the first problem as in Figure 7.

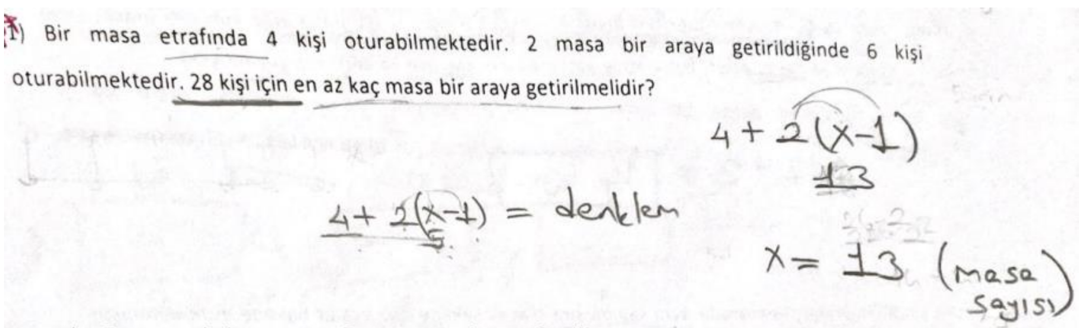

Figure 7. *A Seventh-Grade Student's Solution Using Computational Representation*

When the solution of the student who answered the first problem with computational representation was examined, it was seen that the student reached the correct answer by setting up an equation by using the variables in the problem, instead of using drawings showing the relationship between the variables in the problem. However, unlike this student, most of the students who used the computational representation type gave wrong answers to the problems. For example, the solution of a seventh-grade student using computational representation for the first problem is given in Figure 8.

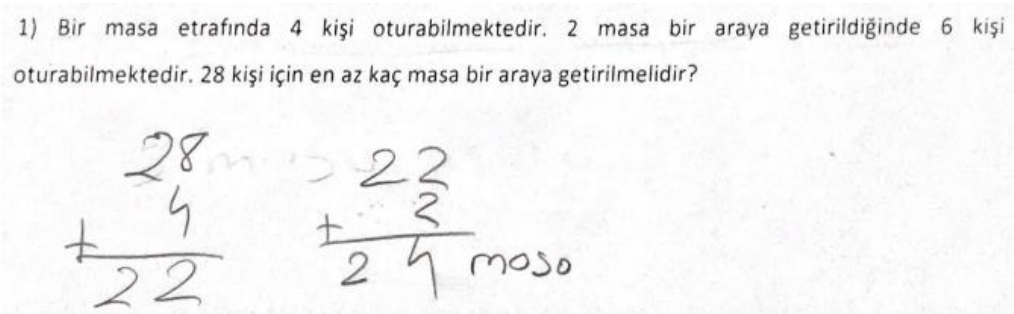

Figure 8. *A Seventh-Grade Student's Solution Using Computational Representation*

When the solution made by the student who answered the first problem with computational representation was examined, it was seen that the student used the numbers in the problem randomly, without understanding the relations of the quantities in the problem and got the wrong answer.

4.2. Findings Regarding the Relationship Between Students' Problem-Solving Performances and the Types of Representations Used

Pearson Correlation Coefficient was used to determine the relationship between the scores of secondary school students from three representation types (schematic, pictorial and procedural) and the total scores they obtained from the Visualization Test. As a result of the analysis, the relationship between the students' total scores and their use of schematic representation is given in Table 2.

Table 2. *Pearson Correlation Analysis Results of Total Test Scores and Schematic Scores*

	Total Test Score	
Total Schematic Score	r	0.473*
	p	0.000
	N	212

*p<0.05

When Table 2 was examined, it was determined that there was a moderate positive relationship between the Visualization Test total scores of secondary school students and their total schematic scores, and this relationship was statistically significant (r=0.473, p<0.05). The coefficient of determination was $R^2 = (0,473)^2 = 0.224$. Based on this information, it could be said that approximately 22% of the variability in the total test scores was explained by total schematic scores (Cohen, 1998).

The Pearson Correlation analysis showing the relationship between the total scores obtained from the Visualization Test and the total pictorial representation scores of the secondary school students participating in the research is given in Table 3.

Table 3. *Pearson Correlation Analysis Results for Total Test Scores and Pictorial Scores*

		Total Test Score
Total Pictorial Score	r	-0.540*
	p	0.043
	N	212

*p<0.05

When Table 3 was examined, it was seen that there was a statistically high level of negative correlation between the Visualization Test total scores of secondary school students and their total pictorial representation scores, and this relationship was statistically significant (r=-0.540, p<0.05). The coefficient of determination was $R^2 = (0,540)^2 = 0.292$. Based on this information, it could be said that approximately 29% of the variability in test total scores was explained by total pictorial scores (Cohen, 1998).

The Pearson Correlation analysis demonstrating the relationship between the total scores obtained from the Visualization Test and the total computational representation scores of the secondary school students participating in the research is given in Table 4.

Table 4. *Pearson Correlation Analysis Results for Total Test Scores and Computational Scores*

		Total Test Score
Total Computational Score	r	-0.399*
	p	0.000
	N	212

*p<0.05

When Table 4 was examined, it was seen that there was a moderately negative relationship between the Visualization Test total scores of secondary school students and their total computational scores, and this relationship was statistically significant (r=-0.399, p<0.05). The coefficient of determination was $R^2 = (0,399)^2 = 0.159$. It could be said that approximately 16% of the variability in test total scores is explained by total computational scores (Cohen 1998).

5. RESULTS AND DISCUSSION

The aim of this study was to examine the types of representations that secondary school students used while solving problems and the relationship between these representation types and students' problem-solving performances. As a result of the analysis, it was concluded that secondary school students mostly used visual representations (schematic and pictorial representations). When we look at the representation types one by one, the most used representation type by the students is the computational representation, which is a non-visual representation type. In addition, the most used visual representation type was schematic representation, and the least used representation type was pictorial representation. Similarly, studies conducted with students on visualization revealed that many students used visual representations while solving problems (Ke & Clark, 2018), and that they also commonly benefited from computational representations (Ives, 2007; Örnek, 2013; Özer, 2020; Van Garderen, 2007). The reason for the conclusion might be that classroom contexts and students' experiences with visual representations in the classroom could shape their preferences and affect how they use different representations to solve problems.

The results of the study indicated that there was a moderate positive relationship between the Visualization Test total scores of secondary school students and their total schematic scores. Moreover, most of the students using the schematic representation gave correct answers to the problems. In the light of these findings, it could be said that students' use of schematic representations increased their problem-solving performance. In a similar way, when the results of the visualization studies in the literature were examined, it was seen that the use of visualization contributed to the problem-solving performance of the students (Anwar et al., 2019; Dockendorff & Solar, 2018; Ergan & Özsoy, 2021; Hegarty & Kozhevnikov, 1999; Ke & Clark, 2018; Kükey et al., 2019; Van Garderen & Montague, 2003; Zeybek & Saygı, 2018). To illustrate, the experimental study of Hegarty and Kozhevnikov (1999) concluded that there was a positive relationship between the use of schematic representation and problem-solving performance. In the same way, Ergan and Özsoy (2021) showed that most of the students who created schematic representation in the problem-solving process gave the correct answer to the problem. Similarly, Van Garderen and Montague (2003) revealed that there was a positive relationship between problem solving success and the use of schematic representations. Likewise, Ke and Clark (2018) argued that schematic representations had a positive relationship with mathematical problem-solving success. With the help of useful visual methods such as graphs, diagrams, or different geometric shapes, abstract concepts in the problems were represented (Sağlam & Bülbül, 2012). Since the term "visualization" refers to the concretization process, which is the result of a high-level cognitive process for determining, explaining, and demonstrating the operation of objects, processes, or events (Sung et al., 2010), it also helps in understanding and solving the problems. Correspondingly, Anwar et al. (2019) argued that the students had an idea about the next action to be taken with the use of schematic representation, and this allowed students to better understand the problem. As a result, schematic representations could help students to identify the underlying mathematical structure of a problem and develop more efficient problem-solving strategies

(Anwar et al., 2019; Dockendorff & Solar, 2018; Hegarty & Kozhevnikov, 1999). Additionally, students may benefit from using schematic representations to connect various mathematical concepts and apply their learning in novel and unfamiliar contexts. According to research (Hegarty & Kozhevnikov, 1999; Verschaffel et al., 1999), employing schematic representations could assist students in applying their understanding of mathematical concepts to new contexts.

The results revealed that there was a statistically high level of negative correlation between the Visualization Test total scores of secondary school students and their total pictorial representation scores. Additionally, none of the secondary school students who tried to produce solutions to problems with the use of pictorial representation could reach the right answer. In the light of these findings, it could be said that students' use of pictorial representations decreased their problem-solving performance. Similarly, when the results of the visualization studies in the literature were examined, it was seen that the use of pictorial representations reduced the problem-solving performance of the students (Ergan & Özsoy, 2021; Hegarty & Kozhevnikov, 1999; Van Garderen & Montague, 2003). For instance, the experimental study of Hegarty and Kozhevnikov (1999) concluded that the use of pictorial representation had a negative relationship with problem solving performance. Similarly, Ergan and Özsoy (2021) examined how the types of visual representations were distributed based on the correct solution of the problem and they found that the majority of the students who were utilized pictorial representations gave the wrong answer to the problem. In a similar way, Van Garderen and Montague (2003) argued that there was a negative relationship between problem solving success and the use of pictorial representations. The reason for the conclusion might be that students used this representation could not realize the relationship between the variables in the problem. Moreover, the usage of pictorial representations might lead to an excess of unnecessary information in the working memory, making it more difficult to focus on and absorb the information that is necessary to solve the problem. In fact, the students used this representation only described in detail the image of items such as person-object presented in the problem instead of realizing relationships in the problem (Hegarty & Kozhevnikov, 1999).

The results showed that there was a moderately negative relationship between the Visualization Test total scores of secondary school students and their total computational scores. Furthermore, most of the students who used computational representations gave wrong answers to the problems. In a similar way, Ergan and Özsoy (2021) revealed that more than half of the students using computational representation gave the wrong answers to the problems. It is important to note that the learning process is quick and permanent when vocally communicated information is made concrete and visualized (Bozkurt, 2012). The reason why the use of visualization accelerates the learning process is that it makes it easier to understand the information, to reveal the meanings and complexity of the rules more easily. Furthermore, visualization can accelerate the learning process by helping students develop deeper conceptual understanding, stronger problem-solving skills, and more meaningful connections between math and the real world. However, the students utilizing computational representation computed the numerical values presented in the problem or the represented by converting them into equations without using visuals (Ergan & Özsoy, 2021).

6. RECOMMENDATIONS

Since it was concluded that secondary school students were more successful in problem solving with the use of schematic representation, students could be directed to use this type of representation more frequently. In addition, it was recommended to support the use of visualization, since students mostly use computational representation but fail in problem solving. For future studies, it was recommended to study the subject of visualization with different sample groups. Moreover, it was recommended to determine the factors that negatively affect students' visualization use and to carry out studies to eliminate these negativities.

7. ABOUT THE AUTHORS

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8. References

Akkan, Y., Akkan, P., Öztürk, M., & Demir, Ü. (2018). Görsel teoremler üzerine matematik öğretmenleriyle nitel bir çalışma [A qualitative study with mathematics teachers on visual theorems]. *Journal of Instructional Technologies and Teacher Education, 7*(2), 56-74. https://dergipark.org.tr/en/pub/jitte/issue/41978/431508

Anwar, R. B., Purwanto, P., As'Ari, A. R., Sisworo, S., & Rahmawati, D. (2019). The process of schematic representation in mathematical problem solving. *Journal of Physics: Conference Series, 1157*(3), 1-5. DOİ: 10.1088/1742-6596/1157/3/032075

Alsina, C., & Nelsen, R. B. (2006). *Math made visual: Creating images for understanding mathematics, 28*. American Mathematical Soc.

Baki, A. (2018). *Matematiği öğretme bilgisi [The knowledge of teaching mathematics]*. Ankara: Pegem.

Başer, N., & Uysal Koğ, O. (2011). Görselleştirme yaklaşımının matematikte öğrenilmiş çaresizliğe ve soyut düşünmeye etkisi [The effect of visualization approach on learned helplessness and abstract thinking in mathematics]. *The Western Anatolia Journal of*

Educational Sciences (WAJES), *2*(3): 89-108. https://dergipark.org.tr/en/pub/baebd/issue/3340/46238

Bogdan, R., & Biklen, S. K. (1997). *Qualitative research for education*. Boston, MA: Allyn & Bacon.

Bozkurt, A. (2012). Matematik öğretmenlerinin matematiksel etkinlik kavramına dair algıları [Mathematics teachers' perceptions of mathematical activities]. *Educatıon and Science, 37*(166), 101-115. http://egitimvebilim.ted.org.tr/index.php/EB/article/view/1367

Budram, J. (2009). *The effects of using visual literacy and visualization in the teaching and learning of mathematics problem solving on grade 6 and grade 7 learners*. Master's Thesis, University of Kwa-Zulu Nata.

Bunar, N. (2011). *Altıncı sınıf öğrencilerinin kümeler, kesirler ve dört işlem konularında problem kurma ve çözme becerileri [The sixth graders' problem posing and solving abilities in sets, fractions and four operations]*. Master's Thesis, University of Afyon Kocatepe.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches*, 2nd Ed. Thousand Oaks, CA: Sage Publishers.

Delibaş, M. E. (2020). *Problem çözmede görselleştirme ve görsel temsil kullanma [Visualization and visual representation in problem solving]*. Master's Thesis, University of Kastamonu.

Dienes, Z. P. (1967). Some basic processes involved in mathematics learning. *Research in mathematics education,* 21-34.

Dockendorff, M., & Solar, H. (2018). ICT integration in mathematics initial teacher training and its impact on visualization: The case of GeoGebra. *International Journal of Mathematical Education in Science and Technology 49*(1), 66-84. https://doi.org/10.1080/0020739X.2017.1341060

Ergan, S. N., & Özsoy, G. (2021). İlkokul dördüncü sınıf öğrencilerinin problem çözme sürecinde oluşturduğu görsel temsillerin incelenmesi [Investigation on the visual representations of 4th-grade students created in the problem solving process]. *Buca Faculty of Education Journal*, *51*, 57-75. https://doi.org/10.53444/deubefd.763452

George, D., & Mallery, P. (2010). SPSS for Windows step by step. A simple study guide and reference. *GEN, Boston, MA: Pearson Education, Inc*, *10*, 152-165.

Gökbulut, Y., Sidekli, S., & Sayar, N. (2013). Dört işlem becerisi nasıl geliştirilir [How to improve the number operations skills]. *International Journal of Turkish Education Sciences, 2013*(1), 31-41. https://dergipark.org.tr/en/pub/goputeb/issue/7323/95836

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of educational psychology, 91*(4), 684-689. https://doi.org/10.1037/0022-0663.91.4.684

Hegedus, S., & Tall, D. (2016). Foundations for the future: The potential of multimodal technologies for learning mathematics. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 543–562). New York: Routledge

Işık, A., & Konyalıoğlu, A. C. (2005). Matematik eğitiminde görselleştirme yaklaşımı [Visualization approach in mathematics education]. *Journal of Kazım Karabekir Education Faculty, 11*(1), 463-471. https://dergipark.org.tr/en/download/article-file/31439

Ives, B. (2007). Graphic organizers applied to secondary algebra instruction for students with learning disorders. *Learning Disabilities Research & Practice, 22*(2), 110-118. https://doi.org/10.1111/j.1540-5826.2007.00235.x

Karasar, N. (2005). *Bilimsel araştırma yöntemleri [Scientific research methods]*. Ankara, Nobel Publishing.

Ke, F., & Clark, K. (2018). Game-based multimodal representations and mathematical problem solving. *International Journal of Science and Mathematics Education*, *18*(1), 103-122.

Korwar, P. (2021). *Introduction to multivariate regression analysis.* http://www.mygreatlearning.com/blog/introduction-to-multivariate-regression/

Kükey, E., Tutak, A. M., & Tutak, T. (2019). Kesirler konusunun görsel materyal ile öğretiminin ilkokul 4. sınıf öğrencilerinin matematik başarı ve tutumlarına etkisinin incelenmesi [An investigation of the effect of the teaching of fractions with visual material on primary school 4th grade students' mathematics achievement and attitudes]. *National Journal of Education Academy,3*(1), 115-125. https://dergipark.org.tr/tr/pub/uead/issue/44894/532949

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. Lester (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 763-804). National Council of Teachers of Mathematics.

Men, O. L., İsmail, Z., & Abidin, M. (2018). Using maths model method in solving pre-algebraic problems among year five students. *In 2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE)*, 222-227. Doi: 10.1109/TALE.2018.8615364

Örnek, T. (2018). *Matematik öğretiminde kullanılabilecek resimler [Pictures to be used in mathematics teaching]*. Master's Thesis, University of Atatürk.

Özer, A. (2020). *Ortaokul 6. sınıf kesirler konusunun görselleştirme ile öğretiminin akademik başarıya etkisinin incelenmesi [Examination of the effect to academic achievement of visualization in the teaching of secondary school 6th grade fractions subject]*. Master's Thesis, University of Kırıkkale.

Rha, I., Sung, E., & Park, S. (2010). Relationships between elementary students' visualization tendency and problem-solving ability. *The Journal of Elementary Education*, *23*(4), 509-534.

Sağlam, Y., & Bülbül, A. (2012). Üniversite öğrencilerinin görsel ve analitik stratejileri [University students' visual and analytic strategies]. *Hacettepe University Journal of Education*, *43*(43), 398-409. http://efdergi.hacettepe.edu.tr/yonetim/icerik/makaleler/378-published.pdf

Stieff, M., & Wilensky, U. (2003). Connected chemistry: Incorporating interactive simulations into the chemistry classroom. *Journal of Science Education and Technology, 12*(3), 285-302. https://www.jstor.org/stable/40188770

Sung, E. M., Leem, J. H., & Kim, S. R. (2010). Affects of visualization tendency of attitude on subject, learning flow, and learning satisfaction through the IPTV based instruction in elementary school. *The Journal of Elementary Education, 23*(3), 293-320.

Tekin, B., & Konyalıoğlu, A. C. (2020). *Matematik öğretiminde görselleştirme [Visualization in mathematics teaching]*. Erzurum: Ertual Publishing.

Wheatley, G. (1998). Imagery and mathematics learning. *Focus on Learning Problems in Mathematics*, *20*(2), 7-16. https://eric.ed.gov/?id=EJ588612

Van Garderen, D. (2007). Teaching students with LD to use diagrams to solve mathematical word problems. *Journal of Learning Disabilities, 40*(6), 540-553. https://doi.org/10.1177/00222194070400060501

Van Garderen, D., & Montague, M. (2003). Visual spatial representation, mathematical problem solving and students of varying abilities. *Learning Disabilities Research & Practice, 18*(4), 246-254. https://doi.org/10.1111/1540-5826.00079

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical thinking and learning, 1*(3), 195-229.

Yıldırım, A., & Şimşek, H. (2018). *Sosyal bilimlerde nitel araştırma yöntemleri [Qualitative research methods in the social sciences]*. Ankara: Seçkin Publishing.

Yılmaz, R., & Argün, Z. (2013). Matematiksel genelleme sürecinde görselleştirme ve önemi [Visualization in the mathematical generalization process and its importance]. *Hacettepe*

University Journal of Education, *28*(28-2), 564-576. http://efdergi.hacettepe.edu.tr/yonetim/icerik/makaleler/202-published.pdf

Zazkis, R., Dubinsky, E., & Dautermann, J. (1996). Coordinating visual and analytic strategies: A study of students' understanding of the group D4. *Journal for research in Mathematics Education*, *27*(4), 435-457. https://doi.org/10.5951/jresematheduc.27.4.0435

Zeybek, N., & Saygı, E. (2018). Apartmanlar oyununun ortaokul matematik öğretmen adaylarının uzamsal görselleştirme yeteneklerine olan etkisi [The effect of skyscrapers game on the spatial visualization ability of prospective middle school mathematics teachers]. *Abant İzzet Baysal University Journal of Education*, *18*(4), 2541-2559. https://doi.org/10.17240/aibuefd.2018.18.41844-504903

Zhang, X., Clements, M. K., & Ellerton, N. F. (2015). Conceptual mis(understandings) of fractions: From area models to multiple embodiments. *Mathematics Education Research Journal*, *27*(2), 233-261. Doi:10.1007/s13394-014-013