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Investigation of the Relationships between Number Sense, Algebraic Reasoning, and Mathematical Achievement of 8th Grade Students

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Abstract: The aim of this study is to investigate the relationship between number sense, algebraic reasoning, and mathematical achievement among 8th grade students. This study aims to describe an existing situation in the research, and so uses a survey model as quantitative research. The Algebraic Reasoning Assessment Tool (ARAT); and number sense test, as developed by Kayhan Altay (2010) were used. This study used the graded score scale, adapted from Marzano (2000), to evaluate students' responses to the ARAT questions. This research was conducted with 290 8th grade students. On evaluation, the results of this research were as expected, because algebraic reasoning is based on number concept and number sense. Accordingly, in order for students to be able to think and reason using algebra, it is necessary that they learn algebra without any problems and they understand the basics of algebra well. On the other hand, arithmetic knowledge is shown to be the basis of algebraic learning. On examination, the definitions of algebra state that it is a generalized form of arithmetic. Arithmetic knowledge is related to a complete understanding of the concept of number. It is concluded that, a student who fully learns the concept of number may also has a developed number sense. This study showed that the algebraic reasoning ability of students with a high number sense is high. Besides, it was determined that students often had difficulty using different solution strategies while answering the questions. In addition, it is concluded that the students' number sense scores were a significant predictor of algebraic reasoning scores, and that 36% of the variance in algebraic reasoning scores could be explained by number sense scores.

Keywords: *Math Education, Number Sense, Algebraic Reasoning, Mathematical Achievement, Secondary School Students.*

1. INTRODUCTION

In a rapidly developing and changing world, individuals need to reason, to know mathematics, and to think mathematically in order to adapt to changes and cope with the problems they face. In addition, individuals who understand, know, and think mathematically have more options when shaping their future (National Council of Teachers of Mathematics

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[NCTM], 2000). Therefore, mathematics is not only a discipline that scientists, engineers, or mathematics teachers should know but a science that every person should know as well. It is emphasized that mathematics is important but learning and teaching mathematics are difficult processes (Moyer & Milewicz, 2002). Whether or not this process runs smoothly depends on effective mathematical reasoning (Bike Kalkan, 2014). Based on mathematical reasoning several mathematical subjects are taught, including numbers, operations, patterns, predictions, and generalizations (Umay, 2003). When handle these topics, two concepts that mathematics educators have frequently dealt with in recent years become prominence: Number sense and algebraic reasoning.

Number sense is based on an understanding of the various relationships among numbers and operations and using these relationships in a flexible way. Yang (2003) describes the number sense as an individual’s skill to gain a good knowledge of numbers, operations, and the relationships between them, as well as their ability to use this knowledge flexibly regarding problems encountered in daily life. Greeno (1991) also refers to number sense according to three different components: Being able to consider different situations by flexibly (flexibility in numerical calculation), the ability to estimate (numerical estimation) while making various calculations; and making inferences from quantitative data (inference with quantitative reasoning). As it can be understood from these definition and classification, number sense coincides with reasoning, thinking, and decision-making skills.

Vance (1998) defines algebra as one of the ways of mathematical thinking. Vance’s definition reveals the concept of algebraic thinking, which also includes the meaning of algebra itself. Algebraic thinking involves the representation of mathematical structures and situations using algebraic symbols, and the use of models to represent quantitative relationships, and to understand functions (NCTM, 2000). Algebraic reasoning forms the roots of algebraic thinking and, significantly, the importance of algebraic reasoning has been emphasized over the last two decades (Kaput, 1995; House, 1999; Russell, 1999; NCTM, 2000; The National Assessment of Educational Progress, [NAEP], 2002). According to Swafford and Langrall (2000), algebraic reasoning is defined as operating an unknown amount, as known quantity. Although the importance of algebraic reasoning is emphasized in recent studies, it is stated that most students do not perform well in algebra courses (Blume & Heckman, 2000). Indeed, Cooper et al. (1997) state those difficulties of students’ experience in algebraic thinking and algebra stem from their lack of understanding of arithmetical structures and relationships. Because the roots of algebra are based on arithmetic (Kieran, 1992; Linchevski, 1995; Van Amerom, 2002), and arithmetic provides algebraic opportunities for symbolization and generalizations, which are necessary for algebraic reasoning (Akkan et al., 2011).

According to some researchers, lack of number sense is seen as an important obstacle for individuals when learning mathematics (Ekenstam, 1977; Yang, 2007). This situation shows how important number sense is in mathematics education (Yang & Wu, 2010). In addition, it is stated that those students who have developed number sense fully comprehend the concept of numbers, what the number is, flexible calculations, and the relationships among operations (Sowder, 1992; Dossey, 1997; Yang & Wu, 2010). Similarly, one of the factors often stated as facilitating students' learning of algebra is that their arithmetic knowledge should be well structured (Booth, 1988; Herscovics & Linchevski, 1994; Kieran, 1992; Sfard, 1995; Wagner 1983). As a matter of fact, Wagner (1983) stated that the reason why students have difficulty in understanding algebraic operations and structures in learning algebra often results from the fact that they are unable to fully understand the concept of number, which is the basis of arithmetic. Students construct their algebraic learning based on their learning and experience about arithmetic (Booth, 1988; Kieran, 1992). Therefore, it may not be possible for someone who has not fully learned arithmetic, the concept of number, the basis of arithmetic to learn algebra. Cooper et al. (1997) state that students' lack of expressing arithmetic construct and relational representations causes students to experience difficulties when learning algebraic thinking and algebra.

In consideration of explanations given above, this study is trying to explain relationship between number sense, algebraic reasoning and mathematics achievement, while also determining whether number sense and algebraic reasoning to predict mathematics achievement in a meaningful manner.

2. LITERATURE REVIEW

2.1. Algebra and Algebraic Reasoning

Several different definitions of algebra, which originated in 1800 BC (Kaya & Keşan, 2014), exist within the literature. In general terms, algebra describes them by placing meaning on numbers, symbols, and unknowns; organizes mathematical relationships; acts as a tool of thought regarding the transition from arithmetical to abstract concepts; and provides in mathematical language's transforming into equations (Kaya, 2015). Kieran (1992) describes algebra as a tool for calculating with symbols, rather than simply representing numbers and symbols. Algebra is a mathematical language used to explain ideas in mathematics and other disciplines (Sutherland & Rojano, 1993; Usiskin, 1997). According to the best of our knowledge algebraic thinking and algebraic reasoning are used with the same meaning in the literature (Bike Kalkan, 2014). Algebraic reasoning is a way of thinking that includes in the most important skills for mathematics, such as reasoning, understanding variables, explaining symbols, working with models, and making transformations between representations (Kaf, 2007). Kaput (1998) considers algebraic reasoning according to three dimensions: generalized arithmetic, functional thinking, and generalization and justification. The relevant sub-dimensions of these are given below.

Generalized Arithmetic:

• *Exploring properties and relationships of whole numbers,*

• *Exploring the properties of operations on whole numbers,*

• *Exploring equality as expressing a relationship between quantities,*

• *Algebraic treatment of number,*

• *Solving missing number sentences.*

Functional Thinking:

• *Symbolizing quantities and operating with symbolized expressions,*

• *Representing data graphically,*

• *Finding functional relationships,*

• *Predicting unknown circumstances using known data,*

• *Identifying and defining numerical and geometric patterns.*

Generalization and Justification:

• *Using generalizations to solve algebraic tasks,*

• *Justification, proof, and testing conjectures,*

• *Generalizing a mathematical process.*

Lawrence and Hennessy (2002) define algebraic reasoning as the ability to predict and interpret situations and events using mathematical language. Moreover, algebraic reasoning not only includes algebra studies, but also the use of multiple representations and the ability to solve problems that require thinking and reasoning (Çelik, 2007). In other words, algebraic reasoning is not only used to solve algebraic questions, but also in many other areas ranging from solving non-routine problems that may be encountered in our daily lives, to solving problems in different disciplines. The development of algebraic thinking skills in students is closely related to their learning of algebra well by creating a good background in algebra (Yenilmez & Teke, 2008). Therefore, developments in the field of algebra also support and influence the development of algebraic reasoning.

2.2. Number Sense and Number Sense Components

Number sense is hard to define but easy to understand (Case, 1998), and so it is very unlikely for two researchers to offer the same definition (Gersten et al., 2005). The broadest definition of number sense considers it be a person's sense of numbers and operations (McIntosh et al., 1992; Yang, 2003). This sense is used flexibly when developing strategies that are essential for complicated problems and making mathematical judgements (Burton, 1993; Reys & Yang, 1998). As number sense deals with numbers, operations, and the relationships among these concepts, as applied to daily life situations, the relationship between math and everyday life is thereby easier to understand. Furthermore, number sense helps foster flexible, creative, and effective mathematical thinking (Yang & Wu, 2010).

Different definitions of number sense are given in literature (Hope, 1989; Howden, 1989; Gersten & Chard, 1999; Greeno, 1991; McIntosh et al., 1992; Reys et al., 1999; Yang, 2003; Berch, 2005). According to Hope (1989), number sense, in the most general meaning, is defined as the ability to make predictions about the use of numbers, detect errors in arithmetic, determine number patterns, and choose effective calculation paths. Howden (1989) states that number sense relates to the natural understanding and intuition of each child. As a matter of fact, Gersten and Chard (1999) stated that every child gains such an understanding and intuition as a result of environment interactions before they start kindergarten; however, not all children have the level of number sense. Gersten and Chard (1999) expressed: “For example, a child may know that 8 is 3 more than 5 but if number sense is improved, he/she can show why 8 is greater than 5 by developing a strategy with the help of his/her fingers or blocks.”

Number sense is a complex process involving certain connections and skills related to numbers. Accordingly, it is more helpful to examine the skills demonstrated by individuals during this process for fully understanding the concept of number sense (Burns, 2007). Considering these skills, number sense as stated in literature divided into several sub-components (Resnick, 1989; McIntosh et al., 1992; Sowder, 1992; Markovits & Sowder, 1994; Yang, 1995, 2002; Reys et al.,1999; Yang et al., 2008b; Li & Yang, 2010; Yang & Tsai, 2010; Yang & Wu, 2010). As was the case regarding definitions for number sense definitions of number sense, different theoretical frameworks relate to the components of number sense itself (Greeno, 1991; McIntosh, Reys & Reys, 1992; Reys et al, 1999). Greeno (1991) considered number sense as comprising three components: *numerical prediction, quantitative reasoning and inference, and flexibility in numerical calculation.* For example, expressing “25×48 as $(\frac{100}{4}) \times 48$, $100 \times \frac{48}{4}$ and 100×12” means that the calculation requires a high-level of skill. Reys et al. (1999) considered number sense according to six components. The classification of number sense components as by Reys et al. (1999) is given below in Table 1.

Table 1. *Number Sense Classification by Reys et al. (1999)*

Number Sense Component	Sample
Understanding the meaning of numbers.	How do you compare 2/5 to 1/2?
Understanding and using number notations	Show different solutions representing 2/5.
Understanding the meaning and effect of operations	750 ÷ 0.98, greater than or less than 750 manager? How do you know?
Usage and meaning of synonyms	Are 70 ÷ 0.5 and 70 x 2 equal? How do you express it?
Mind processing, written processing and counting, and flexible trading strategies for calculator use	By using your knowledge of numbers and operations, 6 x 98 can you do it in mind?
Benchmarking (reference) point	How do you estimate the height of a large object? Do you use a “measurement reference” or “fulcrum”?

Yang and Wu (2010) emphasize several important reasons for learning number sense; i) to provide individuals with effective, flexible, logical, and purposeful ways of thinking; ii) to understand quantities, numbers, procedures, and the ability to apply these integrally in daily life; iii) because number representations and mathematical thinking of adults depend on number sense. Dossey (1997) states that individuals with improved number sense can develop models for problems involving numerical relationships and real-life data. As number sense has a central role in associating mathematics with real life (Charles & Lester, 1984) it is therefore important that individuals have the ability of number sense.

2.3. Purpose of the Study

In several researches on algebraic reasoning (Kieran & Chalouh, 1993; Kaya & Kesan, 2014; Kaya, Kesan, Izgiol & Erkus, 2016; Nathan & Koellner, 2007), it is stated that algebraic reasoning is essential for the achievement of mathematics, and that it should be employed in the most efficient solutions to becoming successful in mathematics. Similarly, many studies investigating the relationship between number sense and mathematics achievement (Yang, Li & Li, 2008; Harç, 2010; Kayhan Altay, 2010, Mohamed & Johnny, 2010; Şengül & Gülbağcı, 2012) state that a positive relationship exists between number sense and mathematical achievement. As a matter of fact, Wagner (1983) states that the reason why students have difficulty understanding algebraic operations and structures in learning algebra results from their inability to fully understand the concept of number, which is the basis of arithmetic. Additionally, researches (Herscovics & Linchevski 1994; Kieran & Chalouh, 1993; Sfard, 1995; Sfard & Linchevski, 1994) also state that students' algebraic thinking is structured according to their arithmetical experiences. In this context, it is important to examine the relationship between students' number sense, algebraic reasoning, and mathematical achievement. So, this study aims to investigate the relationship between number sense, algebraic reasoning, and mathematics achievement among 8^{th} grade students. Accordingly, the answers to several research questions were thereby sought out to realize this aim:

1. What is the level of the number sense of 8^{th} grade students?

2. What is the level of algebraic reasoning of 8^{th} grade students?

3. Is there a significant relationship between number sense, algebraic reasoning, and mathematics achievement of 8^{th} grade students?

4. Do number sense and algebraic reasoning of 8^{th} grade students significantly predict their mathematics achievement?

3. METHODOLOGY

3.1. Research Model

The purpose of this study is to investigate the relationship between number sense, algebraic reasoning, and mathematics achievement among 8^{th} grade students. For this reason, the survey model which is among descriptive research methods. The relational survey model helps determine research patterns that determine the existence and degree of the relationship between or among two or more variables (Karasar, 2009). In this context, the algebraic reasoning assessment tool (ARAT) and the Number Sense Test (NST) were applied to 8^{th} grade students to find out whether a significant relationship exists between the achievement scores of the students.

3.1.1. The Study Content

In the Mathematics Curriculum (2018) in Turkey, achievements related to the "algebra" learning field are first included in the 6th grade. At this grade level, students are expected to find the desired term in number patterns and make sense of algebraic expressions. In the 7th grade; It appears as equality and equation in algebraic expressions. At this grade level, students are expected to perform addition and subtraction operations in algebraic expressions, understand the concept of equality, and solve first-order equations with one unknown and related problems. In the 8th grade, the area of Algebra learning is given much more coverage. At this level, algebraic expressions and identities, linear equations and inequalities are covered. Students are expected to understand algebraic expressions and identities and to factor algebraic expressions. In addition, examination of the linear relationship between two variables and equation solutions are included. Secondary school algebra subjects end with the examination of inequalities with one unknown. However; the algebra learning area is closely related to the "numbers and operations" sub-learning area in 1st, 2nd, 3rd and 4th grade levels. Therefore; Its foundations are laid in primary school. There is no special sub-learning area related to "number sense" in the mathematics curriculum; It mostly appears under the subheading of "numbers and operations" with concepts such as "estimation" and "approximate value" (Ministry of Education [MONE], 2018).

3.2. Participants

On examination of the Mathematics Curriculum (Ministry of Education, 2018), goals related to algebra first take place in 6th grade, but that algebra learning is given much more space in the 8th grade. According to this context, it was decided that research be conducted with eighth grade students. Because students at the eighth grade level are expected to have acquired competencies such as algebraic reasoning and number sense skills based on arithmetic thinking. Therefore, it is important to determine whether students have acquired these skills or not. Therefore, in the selection of the participants in the study, the fact that the students were educated in the eighth grade was accepted as a criterion. The reason for choosing eighth grade students is that it is important to determine the number sense and algebraic reasoning skills of the students who would study at the high school level at the end of secondary school. For this reason, purposeful sampling method was preferred in the selection of the participants. Because purposeful sampling is a technique widely used in qualitative research for the identification and selection of information-rich cases for the most effective use of limited resources (Patton, 2002). Accordingly, research was conducted with 290 8th grade students in six different public schools in the Western Black Sea Region throughout the 2018/2019 academic year in Turkey. In the selection of the participants, in terms of being both easily accessible and applicable; convenience sampling method, which is one of the types of purposeful sampling, was used.

3.3. Data Collection and Data Collection Tool

3.3.1. The Algebraic Reasoning Assessment Tool (ARAT)

A quantitative research method was adopted for this research. Alternatively, the ARAT (Appendix 1), which comprises a total of 18 items-12 of which are open-ended and six of which are multiple-choice was used in line with the algebraic reasoning sub-dimensions of Kaput (1998), which aims to determine students' algebraic reasoning as a data-collection tool through examining native and foreign literature. Following the pilot study, it was decided that six items would be excluded from the test by taking into consideration the item difficulty index value (items that are very difficult or easy) of those items included in the ARAT. For example; *"Ali's weekly earnings are 20 Turkish liras (TL). Ali receives an additional 2 TL for each hour of overtime work. The letter "s" indicates the hours Ali worked overtime. The letter "k" indicates Ali's total money earnings. Write an equation showing the relationship between "s" and "k"*. The question was very difficult for the students. Therefore, it was decided to exclude this question from the test. In addition, in order for students to answer the test within a lesson hour; it was decided to exclude similar questions in the same sub-dimension from the test. For example; one of the two similar questions in the *"finding functional relationships"* sub-dimension was excluded from the test. This test was then applied to 72 students. The first form comprises 24 items, and so a final, 18-item version, of the ARAT was created. The Cronbach's alpha coefficient of ARAT was calculated as .87. Özdamar (1999) states a reliability coefficient between .60 and .90 as being very reliable. In this context, the scale is reliable. The items in each dimension of ARAT are given in Table 2.

Table 2. *Dimensions and Sub-dimensions of Items in ARAT*

Algebraic Reasoning Dimensions and Sub-Dimensions (Kaput, 1998)	**Substances in Test**
1) Generalized Arithmetic *• Exploring properties and relationships of whole numbers* *• Exploring the properties of operations on whole numbers* *• Exploring equality as expressing a relationship between quantities* *• Algebraic treatment of number* *• Solving missing number sentences*	1, 3, 7, and 8
2) Functional (functional, practical) Thinking *• Symbolizing quantities and operating with symbolized expressions* *• Representing data graphically* *• Finding functional relationships* *• Predicting unknown states using known data* *• Identifying and defining numerical and geometric patterns*	2, 4, 5, 6, 10, 11, 12, and 18
3) Generalization–Justification *• Using generalizations to solve algebraic tasks* *• Justification, proof, and testing conjectures* *• Generalizing a mathematical process*	9, 13, 14, 15, 16, and 17

3.3.2. Number sense test (NST)

The 17-item NST, which was developed by Kayhan Altay (2010), was used to measure students' number sense. The NST (Appendix 2) comprises three dimensions: flexibility in the calculation, conceptual thinking in fractions, and reference point. The Cronbach's Alpha coefficient of the test is 0.86. A total of 17 items were included in the NST, 13 of which were open-ended and four of which were multiple-choice. Students were asked to make a statement about the solution of each item. The items corresponding to each dimension of the NST can be seen in Table 3.

Table 3. *Dimensions and Numbers of Items in NST*

Number Sense Test Sub-Dimensions (Kayhan Altay, 2010)	**Substances in the Test**
1) Flexibility in calculation *• Use numbers flexibly* *• Choose the most practical and effective solution strategy* *• Ability to make a practical solution without long-term operations*	1, 3, 4, 6, 7, 8, 10, and 13
2) Conceptual thinking in fractions *• To be able to understand the fraction, part of a whole, a place on the number line, part of the natural numbers* *• To be able to express the magnitudes represented by fractions in models*	11, 12, 14, and 15.
3) Benchmarking (reference) point *• Deciding the point of comparison and use this strategy to facilitate the problem.*	2, 5, 9, 16, and 17.

3.4. Data analysis

For this study, descriptive and inferential analysis was conducted on those data obtained from the ARAT. These data were summarized and interpreted in accordance the previously determined themes, and by examining cause-effect relationships with a number of results (Yıldırım & Şimşek, 2013). In evaluating the questions in the ARAT on the basis of each dimension, the researcher/s used the Progressive Score Measure (PSM), as adapted from Marzano. In this case, the highest point that can be obtained from the ARAT is 72, while the lowest point is determined as 0. In Table 4, PSM is given place to the example of progressive score measure.

Table 4. *Example of Graded Score Scale*

The categories of test items	***Score***	***Observed Student Behavior***
Generalized Arithmetic	0	No judgement no resolution.
	1	In the process of resolution, giving place to arithmetic calculations on algebraic operation, or using the properties of integers but without a solution.
	2	In the process of resolution, reaching the unknown by performing advanced random calculations or getting a partly wrong result.
	3	In the process of resolution, giving place to partial inclusion of algebraic operation of numbers or determining the unknown randomly, but reaching the correct result.
	4	In the process of resolution, using the right algebraic approach, finding the unknown, and reaching the correct result.

The data obtained from NST were evaluated by coding as 1 if the solution was correct (based on sense of number) or 0 if it was based on false or rule based. In this case, the lowest score that can be obtained from SDT is 0 and the highest score is 17. Concerning the determination of variables, percent (%), mean (X), and standard deviation are used, while the relationship between variables is determined by the Pearson Correlation Coefficient. The relationship between students' number sense and algebraic reasoning is accounted with Pearson Moments Conjugate Correlation Coefficient Technic.

Descriptive statistics analysis was carried out to find answers for the research question 1and 2; correlation analysis was conducted and to find the answer for the research question 3; and hierarchical regression analysis was carried out to find an answer to the research question 4. By incorporating independent variables into the analysis in a specific order, hierarchical regression analysis allows for examination of effects on dependent variables.

Additionally; in order to find answers to the third and fourth questions of the research; students' math lesson end of year mean scores were taken as "mathematics achievement score". Therefore, the mathematics course year-end average scores of each student participating in the research were asked from the course teachers. The mean achievement scores of the students were included in the analysis process as the "mathematics achievement score".

Hierarchic regression analysis is different from many other regression analyses, such as multiple regression analysis, as it also allows for the studying of the natural effects of independent variables within a research that involves a large number of independent variables (Pedhazur, 1997). Therefore, hierarchic regression analysis is preferred for answering the 4th research question of this research. Assumptions required for the regression analysis are syndicated; the results of the test of these assumptions are given below.

- The dependent variable must show a normal distribution.

To examine whether data pertaining to the study's dependent and independent variables show normal distribution, we calculated the severity of the complexity of the data. Calculated values are given in Table 5.

For the data to show normal distribution, the strikingly simple values must vary between +1 and -1 (Büyüköztürk, 2012). On examination, Table 5 shows only algebraic reasoning ability from the lower dimensions of the *generalization–justification* dimension except for those variables that appear to be distributed normally .The lowness value of the generalization–justification dimension is below -1, a very small deviation. In some sources, a value of skewness–lowness between 1.5 and +1.5 (Tabachnick & Fidell, 2015) has been taken into account for these data, assuming the normal distribution kept for the data analyses.

Table 5. *Descriptive Statics Related with Dependent and Independent Variables*

Variables	Skewness	Kurtosis	Mode	Median	X
The Math Achievement	-0.458	-0.708	100	73.5	71.94
Algebraic Sum	0.028	-0.694	36	42.5	43.94
Generalized Arithmetic	-0.138	-0.742	10	10	9.68
Functional Thinking	-0.559	-0.219	24	23	22.26
Generalization–Justification	0.386	-1.006	26	10	12
Number Sense sum	0.113	0.143	9	7	7.27
Flexibility on Account	0.268	-0.844	1	3	3.02
Cognitive Thinking of Fraction	0.059	0.143	2	2	1.95
Benchmarking (reference) point	0.077	0.554	3	2	2.29

- There must not be any multiple connection problem between independent variables.

In order to test this assumption, the values of variance inflation factor VIF and tolerance were considered. Accounted values are given in Table 6.

Table 6. *The Values of the Multiple Connection Accounted Belonging to the Variables*

		Tolerance	VIF
1	Generalization–Justification	1.000	1.000
2	Generalization–Justification	.546	1.832
	Functional Thinking	.546	1.832
3	Generalization–Justification	.486	2.059
	Functional Thinking	.514	1.947
	Generalized Arithmetic	.641	1.560
4	Generalization–Justification	.465	2.151
	Functional Thinking	.496	2.016
	Generalized Arithmetic	.634	1.578
	Benchmarking (reference) point	.731	1.367
5	Generalization–Justification	.463	2.158
	Functional Thinking	.494	2.024
	Generalized Arithmetic	.625	1.601
	Benchmarking (reference) point	.552	1.813
	Cognitive Thinking of Fraction	.584	1.712
6	Generalization–Justification	.462	2.165
	Functional Thinking	.479	2.088
	Generalized Arithmetic	.621	1.610
	Benchmarking (reference) Point	.524	1.908
	Cognitive Thinking of Fraction	.538	1.858
	Flexibility on Account	.562	1.779

VIF values lower than 10, and values of tolerance are higher than 0.02, are expected (Field, 2009; Kalaycı, 2010). As can be seen in Table 6, VIF values fall between 1.00 and 2.165, while values of tolerance that fall between 0.462 and 1.00 are seen to change. According to these values, there is no problem of connectivity. After analysis of the assumption, it could be seen that no problem existed, and so subsequent data analysis was carried out.

4. FINDINGS

The findings of this research, as obtained from the statistical analysis of the study data carried out herein, are presented and interpreted in this section.

4.1. Students' Number Sense

In this section, correct or incorrect solutions relating to those data obtained from the NST were evaluated in order to measure students' number sense. Table 7 shows the students' mean scores for each item in the lower dimensions, based on the solution methods relating to the NST.

Table 7. *Means related to Items in the Dimensions of NST*

Lower Dimensions	Substances								
Flexibility on Account	**Q1**	**Q3**	**Q4**	**Q6**	**Q7**	**Q8**	**Q10**	**Q13**	**Total**
	0.29	0.31	0.44	0.43	0.51	0.33	0.29	0.42	3.02
Cognitive Thinking of	Q11	Q12	Q14	Q15					Total
Fraction	0.7	0.6	0.32	0.33					1.95
Benchmarking	Q2	Q5	Q9	Q16	Q17				Total
(reference) point	0.67	0.18	0.18	0.57	0.68				2.28
									7.25*

**NST mean score*

Mean values presented in Table 7 correspond to a possible maximum value of 1 for each list item and a possible maximum value of 17 for the scale overall. When the means are examined a low-level of number sense skills was observed among students.

Furthermore, when mean scores for each sub-dimension are evaluated, the total score from the calculation flexibility dimension is calculated at 3.02 points (out of a possible of 8 points) in the conceptional thinking dimension, at 1.95 for fractions (out of a possible of 4 points), and 2.28 points (out of a possible of 5 points) in the comparison dimension.

It has been determined for this context that students can calculate maximum points for the lower dimensions of the NST in the flexibility on account dimension of the maximum points to be taken approximately 40%; cognitive thinking of fraction take about 49% and benchmarking (reference) point take about 46% .Thus, it was determined that the highest mean scores of students according to the NST belong to the conceptual thinking sub-dimension in fractions, while the lowest mean scores belong to the flexibility sub-dimension in computation. Additionally, students examined whether the solutions to each substance in the NST were sense-based or rule-based, and related findings are presented in Table 8.

Table 8. *Frequency and Percentage Results of Each Sub-dimension of the NST*

Flexibility on Account																
	Q1		Q3		Q4		Q6		Q7		Q8		Q10		Q13	
	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%
Number Sense Based	84	29	90	31	128	44	125	43	149	51	97	33	84	29	121	42
Rules Based/Wrong	206	71	200	69	162	56	165	57	141	49	193	67	206	71	169	58

Cognitive thinking of Fractions								
	Q11		Q12		Q14		Q15	
	f	%	f	%	f	%	f	%
Number Sense Based	203	70.0	174	60	94	32.4	95	32.4
Rules Based/Wrong	87	30.0	116	40	196	67.6	196	67.6

Benchmarking (reference) point										
	Q2		Q5		Q9		Q16		Q17	
	f	%	f	%	f	%	f	%	f	%
Number Sense Based	195	67.2	53	18.3	53	18.3	165	56.9	197	67.9
Rules Based/Wrong	95	32.8	237	81.7	237	81.7	125	43.1	93	32.1

On examination of Table 8, it can determined that the students answered with the most number sense skills was given to item Q11 (70%) in the cognitive thinking of fraction; and the items that they answered using the least number sense skills were included in the benchmarking (Reference) point which include item Q5 (18.3%) and Q9 (18.3%). Additionally, this study also examined the ways students used to solve items by utilizing their least and greatest number sense skill, and included relevant insights. Figure 1 below shows the link to Q11, which is the article that students answered most using their number sense.

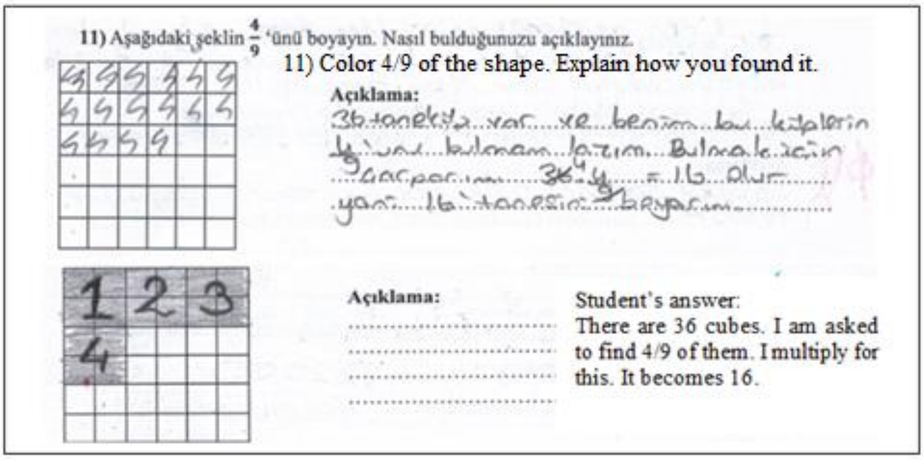


Figure 1. *Solution Strategies of Students Relating to Q11*

On examination of the quote seen in Figure 1, it can be seen that—in trying to find a solution to Q11 (Paint 4/9 of the figure below? Explain how you found it) through the most commonly used methods, such as calculating the value corresponding to a certain amount of the whole and equating the denominator—very few students realized the size of the number, and so were able to solve the question. From this finding it can be determined that students displayed very little tendency to reach the correct answer, based on their number sense.

Additionally, concerning those questions that were answered by students the least, the 5th question, one based on number sense and measurement reference and meaning based on 9th question and size of numbers Figure 2 includes student solutions for Q5 *(Which number should replace a on the following number line? Why is that?)* and Q9 *(Which total is greater than 1? Explain your solution).*

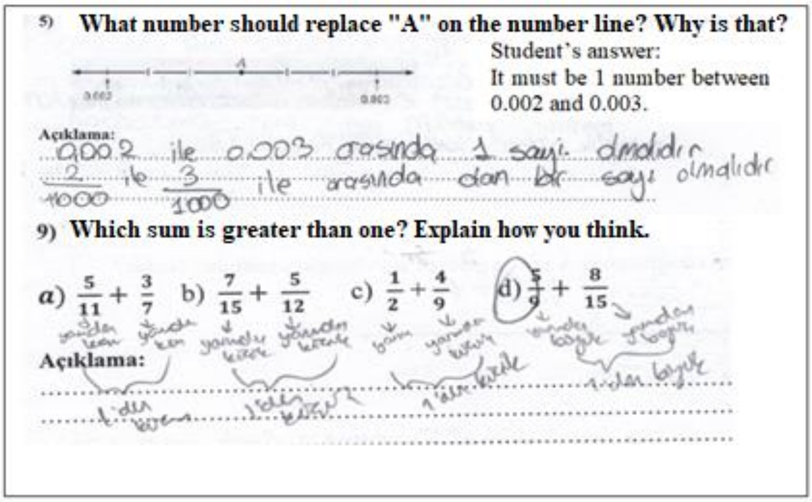


Figure 2. *Student Solutions Strategies Related to Q5 and Q9*

On examination, the quotes seen Figure 2 do not consider the size of reference associated with Q5, and they could not account for the point of a certain size measurement. It was determined that, for some solutions, operations were performed with decimal fractions, but that the student was nevertheless unable to interpret the result. Similarly, most of the correct answers for Q9 *(which total is greater than 1? Explain how you think)*, based on understanding the meaning and the size of the number, are provided through denominator equalization and rule-based resolution (denominator equalization etc.). It has been determined that very few of the solutions are based on number sense, and that a very few Q9-related students achieved their results by using the "being larger than half" strategy, without doing the operation or finding a solution based on number sense.

4.2. Students' algebraic reasoning

The mean scores of 8th grade students for each item in the sub-dimensions of the ARAT in Table 9.

Table 9. *Mean Scores Relating to Substances Involving the ARAT Sub-dimensions*

Dimensions	Substances	X	Total
Generalized Arithmetic	C1	3.4	9.68
	C3	1.6	
	C7	2.0	
	C8	2.6	
Functional Thinking	C2	3.5	22.26
	C4	3.7	
	C5	2.4	
	C6	2.6	
	C10	2.6	
	C11	2.0	
	C12	2.7	
	C18	2.7	
Generalization–Justification	C9	1.9	12
	C13a	1.4	
	C13b	1.3	
	C14	2.0	
	C15	1.9	
	C16	2.1	
	C17	1.4	
			43.94*

**The Mean point of the ARAT*

On examination of Table 9, the values relating to each substance comprise a possible 4 full points, giving a total of 76 overall; the mean is seen to be 43.94 for the ARAT. In this context, it might be said that students' algebraic reasoning level is mean.

In addition, from the algebraic reasoning sub-dimension, the mean number of points given to participating students from the generalized arithmetic size is 9.68 (out of a possible full 16 points), 22.26 (out of a possible 32) at the functional consideration level, and 12 (out of a possible 28) for the generalization–justification level. In this context, it has been determined that, of the maximum points associated with each dimension a student might earn, approximately 60% were given for generalized arithmetic, approximately 69% were given for the functional consideration sub-dimension, and approximately 43% were given for the generalization–justification. In this context, students' highest mean points belong to the functional consideration sub-dimension, while the lowest mean points belong to justification and generalization sub-dimensions.

In addition to this, calculating the frequency and percentage associated with each substance and the ARAT sub-dimension involved can be seen in Table 10.

Table 10. *Percentages and Frequencies of the Items in ARAT Sub-dimentions*

		4 Points		3 Points		2 Points		1 Point		0 Point	
	Substances	**f**	**%**	**f**	**%**	**f**	**%**	**f**	**%**	**f**	**%**
Generalized Arithmetic	C1	209	72.1	9	3.1	63	21.7	8	2.8	1	0.3
	C3	50	17.2	2	0.7	123	42.4	11	3.8	104	35.9
	C7	79	27.2	23	7.9	93	32.1	14	4.8	81	27.9
	C8	167	57.6	3	1.0	33	11.4	20	6.9	67	23.1
Functional Thinking	C2	232	80.0	7	2.4	26	9.0	3	1.0	22	7.6
	C4	243	83.8	12	4.1	30	10.3	0	0.0	5	1.7
	C5	123	42.4	11	3.8	78	26.9	22	7.6	56	19.3
	C6	167	57.6	13	4.5	24	8.3	12	4.1	74	25.5
	C10	121	41.7	21	7.2	97	33.4	11	3.8	40	13.8
	C11	98	33.8	20	6.9	58	20.0	10	3.4	104	35.9
	C12	165	56.9	7	2.4	53	18.3	9	3.1	56	19.3
	C18	183	63.1	3	1.0	20	6.9	6	2.1	78	26.9
Generalization–Justification	C9	51	17.6	12	4.1	138	47.6	30	10.3	59	20.3
	C13a	82	28.3	6	2.1	20	6.9	10	3.4	172	59.3
	C13b	82	28.3	3	1.0	21	7.2	7	2.4	177	61.0
	C14	109	37.6	18	6.2	39	13.4	13	4.5	111	38.3
	C15	97	33.4	6	2.1	72	24.8	10	3.4	105	36.2
	C16	85	29.3	12	4.1	110	37.9	12	4.1	71	24.5
	C17	83	28.6	2	0.7	18	6.2	28	9.7	159	54.8

On examination of Table 10, it can be seen that the highest score was 4 (83%) and the minimum score was Q3 (17%). The highest score of 3 points was seen for Q7 (7%), while the lowest score was seen for Q3 (0.7%) and Q17 (0.7%). Similarly, Q9 (47%) had the highest score, of 2 points, while the most infrequently answered questions were found to be Q13b (6%) and C18 (6%). The highest score of 1 full point was seen for (Q9) (10%), while the lowest was seen for Q4 (0%). Concerning zero points, indicating that students cannot make any judgments and cannot develop solutions, the highest score was seen for Q13b (61%) and the lowest score was seen for Q1 (0%). In this context, when the scores obtained from each item in the ARAT were evaluated, it was found that the distribution of points did not concentrate in a single dimension, and that they scored the highest or the lowest for each item in each dimension.

The relevant quotations pertaining to those solutions for the items with the lowest and highest mean scores regarding ARAT sub-dimensions are given in this section. When Table 9 is examined, the lowest mean score in the generalized arithmetic sub-dimension can be seen to be C3 (1.6 points); and the highest mean score was C1 (3.44 points). In Figure 3, excerpts of C3 are given (Cenk teacher followed the steps below to solve the relevant equation. However, he made mistakes in these steps. Find out at which step he went wrong.) In Figure 4, excerpts from C1 (Which cubes have to be moved to bring the balance into balance?) are given.

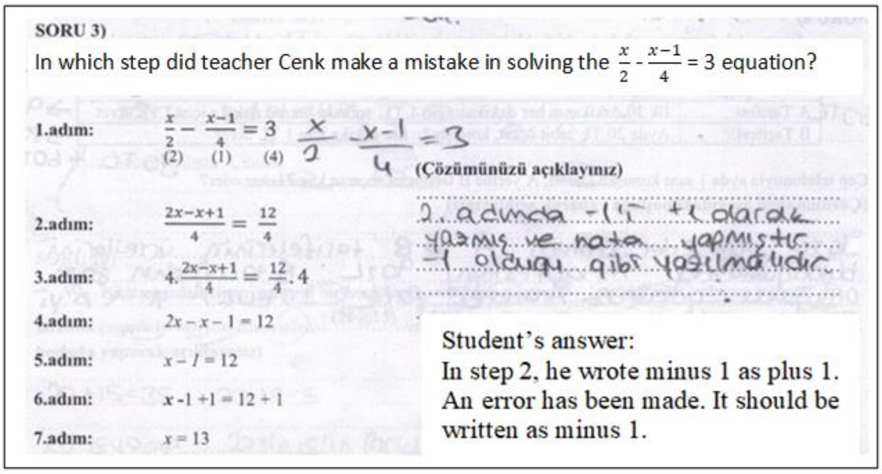


Figure 3. *Student Solution Strategy for C3*

On examination of the quote seen in Figure 3 it can be determined that students generally make mistakes when making transactions using whole numbers, and that they do not consider algebraic operation of numbers. An excerpt from question C1, with the highest mean score, can be seen depicted in Figure 4, in regard to exploring properties and relationships between the quantities of the same sub-dimension.

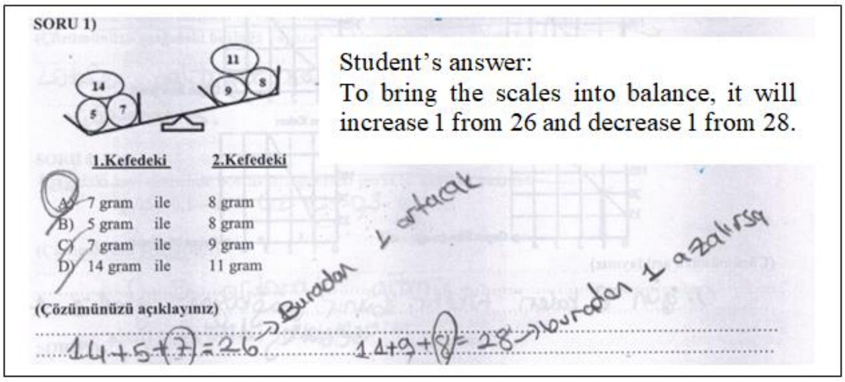
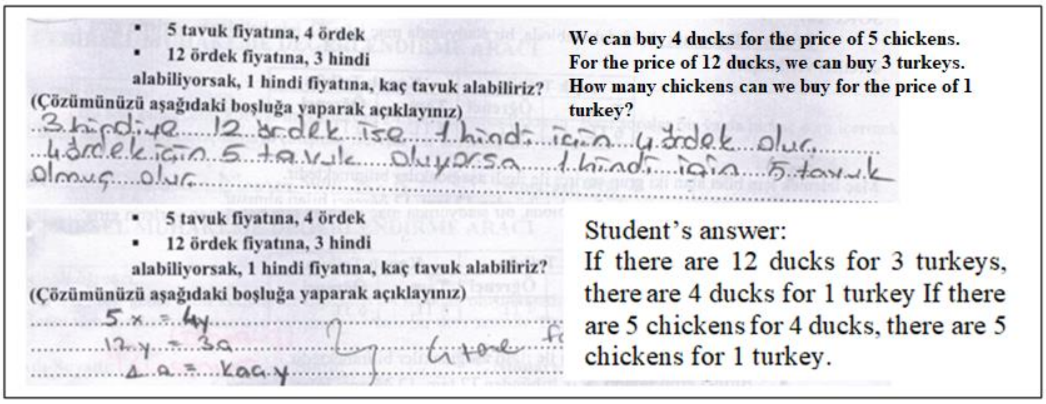


Figure 4. *Student Solution Strategy According to C1*

On examination of the quote seen Figure 3, it can be determined that students can easily explore equality as expressing a relationship between quantities; in order to achieve balance, it is seen that students reached the solution by considering the difference between the quantities in the scale pans. However, students have generally reached the solution by the method of trial and error.

On examination of Table 9 and those items in the functional thinking sub-dimension, it can be seen that the lowest mean score is for C11 (1.99 points), while the highest mean score is for C4 (3.68 points); this includes representing the relationship between the data and the variables graphically. Figure 5 shows quotes given for C11 *(4 ducks are bought for the price of 5 chickens. For the price of 12 ducks, 3 turkeys are purchased. Accordingly, how many chickens can be bought for the price of 1 turkey? Explain your solution).*


- 5 tavuk fiyatına, 4 ördek
- 12 ördek fiyatına, 3 hindi

alabiliyorsak, 1 hindi fiyatına, kaç tavuk alabiliriz?
(Çözümünüzü aşağıdaki boşluğa yaparak açıklayınız)

3 hindiye 12 ördek ise 1 hindi için 4 ördek olur. 4 ördek için 5 tavuk oluyorsa 1 hindi için 5 tavuk olmuş olur.

We can buy 4 ducks for the price of 5 chickens. For the price of 12 ducks, we can buy 3 turkeys. How many chickens can we buy for the price of 1 turkey?

- 5 tavuk fiyatına, 4 ördek
- 12 ördek fiyatına, 3 hindi

alabiliyorsak, 1 hindi fiyatına, kaç tavuk alabiliriz?
(Çözümünüzü aşağıdaki boşluğa yaparak açıklayınız)

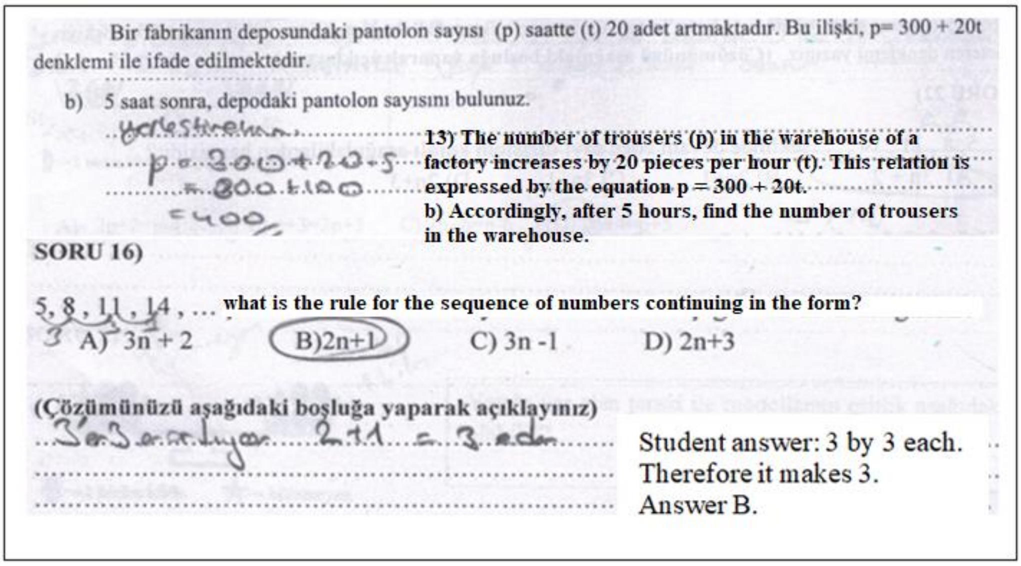
5x = 4y
12y = 3a
1a = kaç y

Student's answer: If there are 12 ducks for 3 turkeys, there are 4 ducks for 1 turkey If there are 5 chickens for 4 ducks, there are 5 chickens for 1 turkey.

Figure 5. *Student Solution Strategies for C11*

On examination of the quote seen in Figure 5, it can be determined that some students reached the solution by making logical inferences, taking into consideration what was given and what was asked in regard to the problem. Among some solutions, it was determined that students tended to find the functional relationship between the variables and destroy the unknowns. However, it was seen that students experienced difficulty responding to those kinds of question that required them to reach the desired result by considering the relationship between different variables; generally, students did not make any judgments about the question.

On examination of Table 9 concerning those items in the generalization–justification sub-dimension, the lowest mean score can be seen for C13b (1.33 points), while the highest mean score can be seen for C16 (2.1 points). Quotations for Q13b *(The number of trousers in a factory warehouse increases by 20 per hour. This relationship is shown by the equation p = 300+ 20t. Accordingly, find the number of trousers in the warehouse after 5 hours*) and Q16 *(Which of the following is the general rule of the sequence of numbers continuing as 5,8,11,14 ...?)* are included in Figure 6 below.


Bir fabrikanın deposundaki pantolon sayısı (p) saatte (t) 20 adet artmaktadır. Bu ilişki, p= 300 + 20t denklemi ile ifade edilmektedir.

b) 5 saat sonra, depodaki pantolon sayısını bulunuz.

yerleştirelim
p = 300 + 20 · 5
= 300 + 100
= 400

13) The number of trousers (p) in the warehouse of a factory increases by 20 pieces per hour (t). This relation is expressed by the equation p = 300 + 20t.
b) Accordingly, after 5 hours, find the number of trousers in the warehouse.

SORU 16)

5, 8, 11, 14, … what is the rule for the sequence of numbers continuing in the form?

A) 3n + 2 B) 2n+1 C) 3n -1 D) 2n+3

(Çözümünüzü aşağıdaki boşluğa yaparak açıklayınız)

3'er 3'er artıyor 2.1.1 = 3 eder

Student answer: 3 by 3 each. Therefore it makes 3. Answer B.

Figure 6. *Student Solution Strategies for Q13b and Q16*

On examination of the excerpts given in Figure 6, it can be determined that only a few students reached the solution by using the method of substituting the value of the unknown for the solution to Q13b; the majority of students did not make any judgments, and not answer the question.

The fact that most students left the response to this question blank in the ARAT indicates that they have difficulty responding to open-ended questions in terms of generalization and justification. Despite C16 having the highest mean score, it was found that students had difficulty in generalizing a mathematical process and generally tended to reach the right answer by eliminating options.

4.3. The Relationship Between Number Sense, Algebraic Reasoning and Mathematical Achievement

Correlation analysis was conducted to test the relationship between the students' number sense, algebraic reasoning, and mathematical achievement. The relationship between the NST sub-dimensions, ARAT sub-dimensions and Math Achievements was calculated by Pearson Correlation Coefficient. These calculated values are given in Table 11.

Table 11. *Correlation Analysis Results*

	1	**2**	**2-a**	**2-b**	**2-c**	**3**	**3-a**	**3-b**	**3-c**
The Achievement of Math	1	.557**	.489**	.507**	.461**	.599**	.505**	.528**	.487**
Algebraic Reasoning		1	.737**	.878**	.918**	.600**	.530**	.472**	.518**
a-Generalized Arithmetic			1	.529**	.564**	.461**	.404**	.383**	.378**
b-Functional Thinking				1	.674**	.541**	.492**	.410**	.458**
c-Generalization-Justification					1	.532**	.459**	.420**	.474**
Number Sense						1	.877**	.826**	.822**
a-Flexibility on Account							1	.560**	.551**
b-Cognitive Thinking of Fraction								1	.618**
c-Benchmarking (reference) point									1

**p<.01

On examination of the findings presented in Table 11, it can be seen that the correlation coefficient between NST achievement scores and ARAT achievement scores is r=.60; the correlation coefficient between NST and mathematics achievement scores is r=.59; and the correlation coefficient between the ARAT and mathematics achievement scores is r=.55. In this context it can be seen that a moderate and positive relationship exists between number sense, algebraic reasoning, and mathematics achievement.

Hierarchical regression analysis was conducted to test the predictive power of the mathematical achievements of the students' number sense sub-dimensions and algebraic reasoning sub-dimensions. First, algebraic reasoning sub-dimensions and number sense sub-dimensions were added to this analysis. The findings obtained as a result of the analysis are presented in Table 12.

Table 12. *Results of Hierarchical Regression Analysis (n=290)*

		β	Std. Error	Std. B	t	F
1	Constant	58.479	1.840		31.774**	
	Generalization–Justification	1.122	.127	.461	8.826**	77.892**
	Model: R^2 = ,21					
2	Constant	41.440	3.664		11.309**	
	Generalization-Justification	.535	.165	.220	3.248**	56.661**
	Functional Thinking	1.082	.204	.359	5.301**	
	Model: R^2 = .28 ΔR^2 = .05 p<0.07					
3	Constant	35.284	3.798		9.29**	
	Generalization-Justification	.281	.169	.116	1.665	
	Functional Thinking	.858	.204	.284	4.211**	47.168**
	Generalized Arithmetic	1.466	.324	.273	4.526**	
	Model: R^2 = .33 ΔR^2 = .05 p<0.048					
4	Constant	33152	3.672		9.027**	
	Generalization-Justification	.109	.166	.045	0.660	
	Functional Thinking	.675	.199	.223	3.389**	
	Generalized Arithmetic	1.300	.313	.242	4.156**	44.619**
	Benchmarking (reference) point	4.321	.863	.272	5.006**	
	Model: R^2 = .39 ΔR^2 = .05 p<0.054					
5	Constant	33.201	3.550		9.352**	
	Generalization-Justification	.066	.160	.027	0.413	
	Functional Thinking	.617	.193	.205	3.202**	
	Generalized Arithmetic	1.133	.305	.211	3.721**	42.393**
	Benchmarking (reference) point	2.138	.961	.135	2.225*	
	Cognitive Thinking of Fraction	4.276	.934	.269	4.580**	
	Model: R^2 = .43 ΔR^2 = .05 p<0.042					
6	Constant	33.971	3.533		9.615**	
	Generalization–Justification	.045	.159	.018	0.282	
	Functional Thinking	.534	.194	.177	2.752**	
	Generalized Arithmetic	1.077	.303	.201	3.556**	
	Benchmarking (reference) point	1.603	.977	.101	1.641	36.948**
	Cognitive Thinking of Fraction	3.614	.964	.227	3.749**	
	Flexibility on Account	1.453	.594	.145	2..449*	
	Model: R^2 = .44 ΔR^2 = .012 p<0.05					

The dependent variable: the achievement of math; **p<0.01 *p<0.05

On analysis of the results presented in Table 12, it can be seen that the regression analysis shows that the percentage of the independent variables included in the analysis hierarchically is explained by the level pertaining to the dependent variable (level R^2=.39, 5th level R^2=.43, and 6th level R^2=.44). In addition, the F values obtained for all levels were significant for $p<0.01$. In this context, it can be said that the ratios of explaining the variance of mathematics achievement of the sub-dimensions that constitute number sense and algebraic reasoning are significant (Tabachnick & Fidell, 2007). Indeed, R^2=.44 is calculated for the 6th level, where all the independent variables can be found. In this context, 44% of the variance in mathematics achievement scores can be explained by the number sense and algebraic reasoning sub-dimensions.

When the findings obtained from the table 12 are evaluated independently, the last level of the regression analysis is that of the 6th. Since the effect of independent variables on the dependent variable is controlled according to each example, it can be said that the 4 sub-dimensions at this level are a significant predictor of mathematics achievement. These dimensions are functional thinking (β=.17, $p<.01$), generalized arithmetic (β=.20, $p<.01$), conceptual thinking in fractions (β=.22, $p<.01$), and flexibility in calculation (β=.14, $p<.05$). Considering that these sub-dimensions predicted mathematics achievement in a meaningful and positive direction; it can be said that these variables are important for mathematics achievement.

This result, obtained by hierarchical regression analysis, is partially consistent but not consistent with the result obtained by correlation analysis. For example, a significant and positive correlation (r=.46, $p<.01$) between mathematical achievement, and proving and generalizing was obtained from the correlation analysis, which concerns the algebraic reasoning sub-dimension. Such a correlation was not yielded by the regression analysis. The reason for this situation is that, in regression analysis, the relationship between the independent variables included in the analysis different from the correlation is thought to be controlled. As a matter of fact, it is thought that regression analysis yields different reliable results from those of other correlation analyses in which each variable is independent from one another (Tabachnick & Fidell, 2007).

According to the findings obtained in the research, 44% of the variance in mathematics achievement scores of the students in the model at Level 6 in Table 12 is explained by the number sense and algebraic reasoning sub-dimensions; accordingly, it can be concluded that these sub-dimensions are predictors of mathematics achievement.

5. DISCUSSION, CONCLUSION AND SUGGESTIONS

This study examined the relationship between number sense, algebraic reasoning, and mathematical achievement among 8th grade students, and discussed whether number sense and algebraic reasoning were a significant predictor of mathematics achievement. The results obtained in this study were discussed comparatively with those of related researches in the literature, and suggestions were presented accordingly.

On evaluation of the results pertaining to students' number sense, it was determined that no student got a full score and that students' number sense skill was low, according to the number sense test. The item to which students answered correct most when using number sense was understanding the meaning of fractions and expressing fractions using the table and circle models (cognitive thinking of fraction); answered correct least provided concern the flexibility on account and benchmarking (reference) point. In the study of Harç (2010) on 6th grade students and the use of number sense, the component with the most correct answer was found to be the measurement reference; this finding is in contrast to the results of the current research. In addition, it was determined that the students' tendency to reach the correct answer based on number sense was very low, especially in regard to multiple choice questions, which were found to be mostly solved by the method of trial and error. The findings of this study also show that students tend to use rule-based strategies and tended to reach conclusions without reasoning. In fact, this finding is similar to related researches in that a few of the students used strategies based on number sense; students generally reached solutions by adhering to the rules learned in the school, and could not use effective strategies (Akkaya, 2016; Alajmi & Reys, 2010; Alsawaie, 2012; Facun & Nool, 2012; Sing, 2009; Yang, 2005).

In addition, it was determined that students often had difficulty using different solution strategies while answering the questions. These results are similar to those of other studies in the literature (Harç, 2010; Menon, 2004; Singh, 2009; Şengül & Gülbağcı, 2013; Yang, 2003; Yang, 2008). The efficacy of number sense teaching can be shown by students' inability to apply flexible solution methods to the questions within a number sense test, as well as their inability to develop different strategies (Şengül & Gülbağcı, 2013). Accordingly, it seems that more importance should be given to teaching number sense to students in the mathematics curriculum. In addition, Yang et al., (2009) stated that one of the reasons why students' sense of number were low was that teachers did not know how to help students improve their number sense, and thought that the students' number sense skills to be insufficient. Additionally, other related researches (Anghileri, 2006; McIntosh et al., 1997) state that the greatest role in the development of number sense falls under the responsibility of teachers. It is therefore believed that it would be beneficial for the teachers to take the necessary trainings in order to provide number sense to the students.

On evaluation of the results of algebraic reasoning, it was indicated that the highest mean scores of students belong to the functional thinking sub-dimension, and the lowest mean scores belong to the generalization and justification sub-dimensions. The fact that the students chose to leave their responses to this ARAT generalization and justification sub-dimension question blank reveals that students had difficulty answering open-ended questions in terms of generalization and justification. One of the most important reasons can be seen in regard to students' algebra learning experiences throughout their early ages (Kaya et al., 2016). Some researchers state that it is correct to provide students with algebraic reasoning at an early age, but that this only be done using the appropriate methods and techniques (Kieran, 2004; Yackel, 1997). Related research also states that the content of mathematics curriculum is very important during the period from preschool to 12 years of age in regard to algebraic reasoning and algebra learning (Blanton & Kaput, 2005; Jacobs et al., 2007). Therefore, students should be provided with opportunities to develop their algebraic reasoning from early ages.

Regarding the findings of this research, it can be seen that number sense and algebraic reasoning skills are significant predictors of mathematics achievement and that 44% (R2=.44) of the variability among the math achievement scores can be explained by number sense and algebraic reasoning. Concerning cognitive thinking of fraction and the flexibility on account; this research revealed that the generalized arithmetic and functional-thinking dimensions of the algebraic reasoning sub-dimensions significantly predict mathematics achievement. In this context, it can therefore be concluded that a student whose number sense and algebraic reasoning is developed will have a high level of mathematics achievement. As a matter of fact, in the literature algebraic reasoning skill is essential for mathematics achievement, and that algebraic reasoning should be used in the most efficient way for achievement in mathematics (Kaya & Keşan, 2014; Kaya et al., 2016; Kieran & Chalouh, 1993; Nathan & Koellner, 2007). Comparatively, Ross (1998) states that, for those students who have not developed algebraic reasoning, mathematics would remain as a collection of calculations that are developed by following certain rules. Teachers are required to teach students to maximize their understanding of algebra (Kitt & Leitze, 1992). As was found in the current study, a positive relationship exists between number sense and mathematics achievement, and this is shown in related literature (Harç, 2010; Kayhan Altay, 2010, Mohamed & Johnny, 2010; Şengül & Gülbağcı, 2012; Yang et al., 2008).

On evaluation, the results of this research were as expected, because algebraic reasoning is based on number concept and number sense. Accordingly, in order for students to be able to think and reason using algebra, it is necessary that they learn algebra without any problems and they understand the basics of algebra well (Yenilmez & Teke, 2008). On the other hand, arithmetic knowledge is shown to be the basis of algebraic learning. On examination, the definitions of algebra state that it is a generalized form of arithmetic (Van Amerom, 2002). Arithmetic knowledge is related to a complete understanding of the concept of number. A student who fully learns the concept of number can also be said to have a developed number sense.

Considering the findings of the current study, the suggestion is made to focus on learning the concept of number, which is directly related to number sense, in order to make learning algebra easier. In addition, reform of mathematics curriculum in Turkey since 2005 and the effective use of skills such as forecasting and mind-making are among the main objectives of mathematics education; despite this, number sense is not included as a concept (Gülbağcı Dede, 2015). Yang et al., (2009) suggest that teacher's inability to develop a sense of number among their students is a reason as to why student fail to use number sense. Since it is difficult for a teacher who is inadequate in a certain field to explain what should be learned within that field (Ball et al., 2008), it is suggested that teachers should receive training in knowledge pertaining to the relevant subject and pedagogical area. A teacher with a limited understanding of the concept of number sense cannot be expected to improve this ability without its use. Additionally, it will be useful for action research to be conducted on those sub-dimensions of the students' number sense and algebraic reasoning skills in order to overcome related difficulties.

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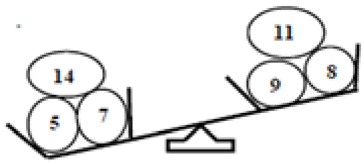
Appendix / Appendices

1) The Algebraic Reasoning Assessment Tool (ARAT)

Dear student,

This test consists of 18 questions. Some questions contain one or more questions. Please try to answer all questions. Your duration is one lesson hour. Good luck.

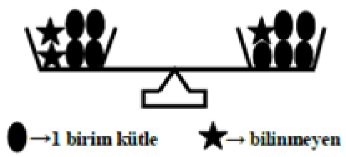
1)



A student wants to balance an equal-armed scale shaped on the side. Accordingly, which masses on the pans should change their places?

	right pan	left pan
A)	7 gr	8 gr
B)	5 gr	8 gr
C)	7 gr	9 gr
D)	14 gr	11 gr

2)



Which of the following could be the equation modeled by the scales on the right?

A) 2p+2=p-4 B) p+3=2p+5 C) 2p+6=p-6 D) 2p+4=p+5

(Explain your solution) ……………………………………………………………………………

3)

$\frac{x}{2} - \frac{x-1}{4} = 3$ Teacher Cenk is solving the equation on the right. However, he made a mistake. Find out which step the teacher made a mistake.

(Explain your solution)
……………………………………………………………
……………………………………………………………
……………………………………………………………

1.step: $\frac{x}{2} - \frac{x-1}{4} = 3$
(2) (1) (4)

2. step: $\frac{2x-x+1}{4} = \frac{12}{4}$

3 step: $4.\frac{2x-x+1}{4} = \frac{12}{4}.4$

4. step: $2x - x - 1 = 12$

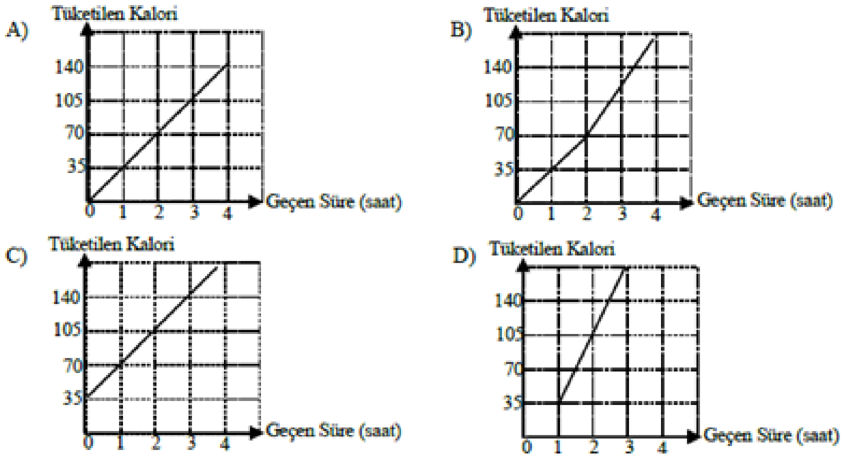
5. step: $x - 1 = 12$

6. step: $x -1 +1 = 12 + 1$

7. step: $x = 13$

4) The table below shows the variation of calories consumed by an athlete over time. Which of the options shows the graph of the table?

Time	1	2	3	4
Calories	35	70	105	140



(Explain your solution) ………………………………………………………………………

5) Two different call tariffs of a mobile operator are given in the table below.

Tariff Type	Tariff Fee
A	1 TL for each minute of the first 30 minutes, 2 TL for each subsequent minute.
B	Fixed fee of 20 TL per month, 1 TL per minute spoken.

If Cemil, who talks on her mobile phone for 1 hour a month, chooses tariff B instead of A, how much profit will she make? (Explain your solution)

6) Write the numbers that should come in the blanks in the number sequence below.

0,05 – 0,1 – 0,15 - ….. - 0,25 – ….- 0,35

(Explain your solution)

7) Problem: The status of İrem, Gül and Hakan's wrist watches according to the correct time is as follows.

- İrem : 2 min. back
- Gül : 6 min. further
- Hakan : 4 min. back

Each of these people, who agreed to meet at a certain time, arrived at the meeting place on time according to their own time. How many minutes are there between the first person to arrive at the meeting place and the last person to arrive?

Solution: If we say x to the correct time

• İrem's watch shows x-2,
• Gul's watch shows x+6,
• Hakan's watch shows x-4.

In this case, Gül will be the first to arrive, and Hakan will be the last to arrive. The algebraic operation that gives the minute difference between two people is as follows.

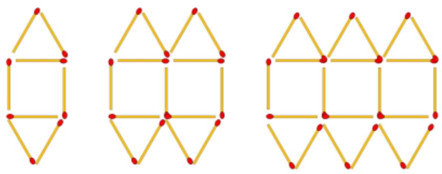
Minute difference = (x+6) – (x-4)
Minute difference = x+6-x-4
Minute difference = 2

Explain by ticking the correct option for this situation..

A) The solution is correct. Because, …
B) The solution is wrong. Because, …

8) According to a new calculation system developed on the computer; When "@" is pressed, it gives the product of the sum of 2 numbers and their difference. According to this; What is the result obtained by the operation ; (20@15) ?
(Explain your solution)

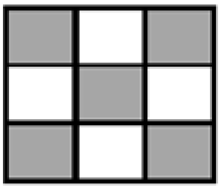
9)



1 2 3

When the above figure is continued; Find the number of matchsticks in figure 26.
(Explain your solution)

10)



The figure on the right is a square made up of 9 equal squares. How many squares are there in total in the figure?

(Explain your solution)

11)

- We can buy 4 ducks for the price of 5 chickens.
- We can buy 3 turkeys for the price of 12 ducks.

How many chickens can we buy for the price of 1 turkey?
(Explain your solution)

12)

A	B	C	D	E	F
1		2		3	
	4		5		6
7		8		9	
	10		11		12

In the table given on the side, the counting numbers are arranged in a certain order. According to which letter does the number 20 come under?

(Explain your solution)

……………………………………………………………
……………………………………………………………

13		…		…	

13)

The number of bagels (s) in a bakery decreases by 30 per hour (t). This relationship is expressed by the equation s = 500-30t.

a) Explain the equation s = 500-30t.

…………………………………………………………………………………………………

b) After 7 hours, find the number of bagels in the oven

……………………………………………………………………………………………………

14)

Şener teacher obtained the 2nd shape by using three of the rectangles he showed in the 1st figure in the classroom. If the perimeter of the second shape is 20 units, what is the value of x?

(Explain your solution)

15)

Problem: In the table below, the entrance fees for the designated places to watch a match in a stadium are given.

A		B	
Adult	Student	Adult	Student
6 TL	4 TL	8 TL	6 TL

The following is known about the two groups of spectators who bought tickets to watch the match.

• The first group of spectators bought tickets for 12 adults and 12 students from the A tribune.

• The second group of spectators bought tickets for 10 adults and 5 students from the B tribune.

Solution:

The wage paid for the first group is found by the equation = (12x6) + (12x4).

The wage paid for the second group is found by the equation = (10x8) + (5x6).

Total money paid = [(12x6) + (12x4)] x [(10x8) + (5x6)] algebraic operation.

Explain by ticking the correct option for this situation.

a) The solution is correct. Because ……………………………………………………..

b) The solution is wrong. Because ……………………………………………………..

16) 5, 8 , 11 , 14 , … Which of the following is the rule for the sequence of numbers?

A) 3n + 2 B) 2n+1 C) 3n -1 D) 2n+3

(Explain your solution)

17) Özlem, who works at a perfume company, mixes a red scent (4x-4) grams, a white scent 3 (y+2) grams, and a pink scent (x+y) grams to create a new scent.

If Özlem mixed 8 grams of red scent and 9 grams of white scent, how many grams of pink scent did she mix? (Explain your solution)

18) A number machine displays the following values against the number keyed in. Find the value corresponding to the question mark.

Input	2	3	4	5	6
Output	11	16	21	26	?

(Explain your solution)

…………………………………………………………………………………………

2) Number Sense Test (NST)

Dear student,

This test consists of 17 questions covering the number sense. Please try to answer all questions. Your duration is one lesson hour. Good luck.

1) How do you solve 0.25 x 16 operation in short way? Show how you did it.

Explanation:………………………………………………………………………………

2) Write a fraction between $\frac{1}{2}$ and $\frac{6}{7}$ .Explain how you found it.

Explanation:………………………………………………………………………………

3) Is the result of 6464 x 0.54 operation greater or less than 3232? Why is that?

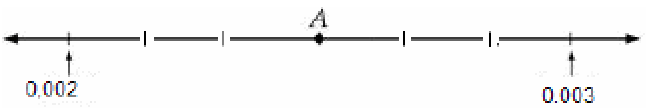
Explanation……………………..…………………………………………………………

4) If 372 – 38 = 334, find the result of 372 – 18 in a short way? Show how you found it.

Explanation:

…………………………………………………………………………………………

5) Which number should replace A on the number line below? Why is that?



Explanation:………………………………………………………………………………

6) Which numbers can be written inside the parentheses to provide the following equation? Explain how you think.

50 + (**…**) : **(…) = 65**

Explanation:………………………………………………………………………………

7) "What is the decimal number 4,358 plus 10?" The solutions of four students for the question are given below. Which road is closest to you? Why is that?

Gökşin'in yolu	İhsan'nın yolu	Mirkan'ın yolu	*Mert'in yolu*
4,358 *+ 10* *14, 358'dir.*	*4, 358* *+ 10* *4, 368'dir.*	*4,358* *+ 10* *4,458'dir.*	*Tam kısımları toplasam yeter.* *4 + 10 = 14* *Cevap 14,358'dir.*

Explanation:
………………………………………………………………………………………

8) How do you do the following in an easy way? Explain how you did it.5 000 032 + 2 000 725 + 1 000 068 – 1 000 725

Explanation:………………………………………………………………………………

9) Which sum is greater than 1? Explain how you think

.*a*) $\frac{5}{11} + \frac{3}{7}$ b) $\frac{7}{15} + \frac{5}{12}$ c) $\frac{1}{2} + \frac{4}{9}$ d) $\frac{5}{9} + \frac{8}{15}$

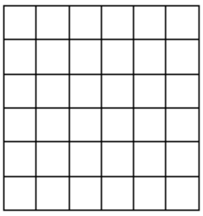
Explanation:………………………………………………………………………………

10) What is the easiest way to find the middle number after sorting the following decimals? Find the number and explain how you found it.

0,10 0,98 0,198 1,3 1,6 1,602 0,835 9,345 0,01

Explanation:
………………………………………………………………………………………

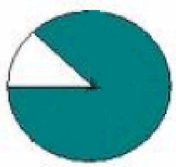
11) Color $\frac{4}{9}$ of the figure below. Explain how you found it.



Explanation:

……………………………………………………………………………
……………………………………………………………………………

12) What range is the number representing the painted area (black part)? Explain how you think.



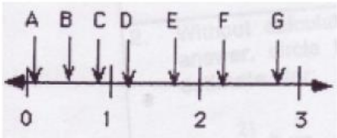
a) **0 to** ¼
b) ¼ **to** ½
c) ½ **to** ¾
d) ¾ **to** 1

13) "The result of 9468 x $\frac{1}{2}$ is greater than the result of $\frac{9468}{\frac{1}{2}}$ "

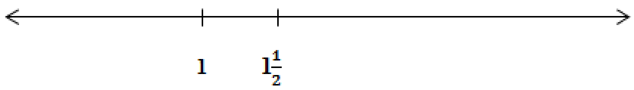
Do you think this statement is correct? Please explain.

Explanation:……………………………………………………………………………..

14) Which letter on the number line corresponds to a fraction whose numerator is slightly larger than its denominator? Explain how you found it.



Explanation …………………………………………………………………………………..



15) Considering the points on the number line given above, insert the fractions $\frac{1}{2}$, $2\frac{1}{2}$ and $\frac{1}{4}$. Explain how you installed it.

Explanation: ………………………………………………………………………………

16) How do you solve 86424 x 500 operation in short way? Show how you think.

Explanation ………………………………………………………………………………

17) Ayşegül teacher asked 60 students in her class about their favorite sports. The table below shows the rates of being liked by sports branches. How do you find out which sport is the most loved by the students in the class? Explain how you think.

MOST FAVORITE SPORTS				
Sports	Football	Basketball	Ping pong	Volleyball
Students	$\frac{2}{5}$	$\mathbf{\frac{7}{12}}$	$\frac{1}{12}$	$\frac{1}{10}$
Explanation:				