

The Effect of Applications related to Augmented Reality in Mathematics Lessons on the Development of Students' Geometric Thinking Levels and Spatial Ability

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Abstract: The aim of this research is to determine whether the use of augmented reality applications in the teaching of the subject of "Geometric objects" in the sixth grade mathematics class has an effect on the development of students' geometric thinking levels and spatial ability. The research was conducted using a pre experimental design, one of the quantitative research methods. The research was carried out with 15 sixth grade students studying in a state secondary school in Aziziye district of Erzurum province in the spring semester of the 2021/2022 academic year, for 15 class hours which each of the lessons consist of 40 minutes. While collecting data in the research, van Hiele Geometric Thinking Test, Middle Grades Mathematics Projects (MGMP) Spatial Ability Test and the Geometric Objects Achievement Test were used. Statistical package program was used in the analysis of the data. As a result of the analysis of the data obtained from the van Hiele Geometric Thinking Test, the MGMP Spatial Ability Test and the Geometric Objects Achievement Test, it was determined that there was a statistically significant difference between the pretest and the posttest in favor of the posttest. It concluded that teaching geometry with augmented reality applications had a positive effect on the development of students' geometric thinking levels and spatial ability, and in addition, it had a statistically positive contribution to their academic achievement. Therefore, in line with these results, it can be said that augmented reality- based teaching applications offer new opportunities for learning- teaching processes.

Keywords: Augmented Reality, Geometric Objects, Mathematics Education, Geometric Thinking Level, Spatial Ability

1. INTRODUCTION

Although mathematics is one of the basic courses of many programs and it is accepted that mathematics questions are decisive in many exams, there is a widespread belief in the world that mathematics course is difficult (Başar et al., 2002; Buyruk -Akıl, 2020; Koparan et al., 2023). In the formation of this belief, there are reasons such as individuals having difficulties in abstract thinking, prejudices and environmental factors. Since some concepts in mathematics, which is a system consisting of abstract concepts, require more abstract thinking skills than others, it is revealed that these concepts are not understood by the student in the process of establishing the connection between the abstract nature of mathematics and

the representations in the student's mind (Delice & Sevimli, 2010). It is seen that the low results of national and international exams such as TIMSS and PISA in our country support the judgment that the abstract nature of mathematics is not understood by the students (Altun & Akkaya, 2014).

Geometry, where the concrete aspect of mathematics stands out most, is an important tool in describing the world and expressing abstract mathematical concepts and relationships (İbili, 2013; Kalay, 2015; Karakuş, 2011). Geometry, which consists of concepts that can be perceived by modeling and proven generalizations, consists of shapes that are not found in nature, mostly within the framework of Euclidean geometry in our schools (İbili, 2013; Karakuş, 2011). Euclidean geometry, which consists of definitions and abstractions, shapes geometry teaching and requires abstract thinking skills. In this sense, one of the geometry subjects that includes concepts that require more abstract thinking skills is geometric objects. The subject of geometric objects is a 3-dimensional subject that requires analytical thinking skills and different perspectives (Özçelik & Semerci, 2016). The biggest problem in learning geometric objects is that students have problems in thinking in 3D and have difficulty in understanding the appearance of geometric objects (Accascina & Rogora, 2006). Showing three-dimensional objects on paper, fitting them into definitions and representing them with diagrams makes us think that we can only imagine geometric objects and structures by reasoning, and it becomes inextricable for students. Therefore, students should be given the opportunity to be successful in teaching geometric objects and concrete materials should be included in teaching (Akın, 2022). In this directions, different teaching methods should be used to embody the teaching of geometric objects and to enable students to think spatially.

Today, with the developing technology, the integration of technology into education is increasing and learning environments are enriched with technology. Many senses are activated through the multimedia provided by technology and it is known that the inclusion of senses in learning environments strengthens learning (Erbaş & Demirer, 2014). Therefore, the use of technological tools in learning environments strengthens learning and the use of technological tools in learning processes is of great importance for the new generation growing with technology. Because technology offers students a learning environment that they can enjoy and students want to teach their lessons with technological materials (Topraklıkoğlu, 2018). At this point, considering the students' attitudes towards mathematics, the use of innovative technological materials is matter in concretizing the geometry subjects at the secondary school level and visualizing them in 3D in a short time, and with it supports the active participation of the students (Hidajat, 2023; Koparan et al., 2023). One of the innovative technologies that help embody the abstract nature of mathematics is Augmented Reality (AR) technology.

Augmented Reality (AR), an appendage of Virtual Reality (VR), can be defined as an interface between the real world and the virtual world; It can also be defined as a natural and realistic human-computer interaction experience where users can interact with real environments (Cai et al., 2014). Although AR is generally used by engineers for product development in our country, today's educators have increased their studies in this field (Erbaş

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& Demirer, 2014). AR technology, which offers concrete interaction opportunities, not only allows students to create their own knowledge structures in educational environments, but also improves spatial ability, which is the ability to visualize objects in 3D in the mind, with the 3D aspect it offers (İbili, 2013; Hidajat, 2023; Koparan et al., 2023). AR applications, with their physical interaction and 3D aspects, make them interesting by creating new interactive environments for mathematics learning that seems boring and ordinary to students (Cahyono, 2018). Radu (2012) states that AR applications increase the cooperation and motivation of students in learning environments and have many positive effects such as easy learning. Akın (2022), on the other hand, argues that augmented reality applications in education enable students to learn easily and effectively in terms of embodying abstract concepts, attract students' attention with the sense of reality it gives, and provide active participation in lessons. In addition, most of the researches state that teaching with augmented reality applications makes it easier for students to understand the subjects by making their learning processes more enjoyable and entertaining (Akçayır, 2017; Aktaş et al., 2017; Chiang et al., 2014; İbili, 2013; Rambli et al., 2013; Sırakaya, 2015; Sırakaya & Seferoğlu, 2016).

In this direction, when the studies on AR technology in mathematics education are examined, it is seen that it is preferred in the teaching of subjects that require students to think abstractly, and it has been observed that AR supported learning environments contribute to the students' discovery process, success and motivation of the relationship between mathematical concepts (Hidajat, 2023; Kulkov et al., 2021; Özdemir & Özçakır, 2019; Widada et al., 2021). In their research, Yanuarto and Iqbal (2022) aim to develop a learning environment with AR applications to improve secondary school students' mathematical spatial abilities on geometric objects. As a result of the research, it was observed that learning environments created with AR had a positive effect on the effectiveness of classroom lessons, and it was stated that AR technologies would make a positive contribution to the development of students' mathematical spatial abilities. Lubis et al. (2022) in the study, which did not aim to investigate the effect of primary school students on their anxiety in mathematics learning, a quasi-experimental design with pre-test-post-test groups, one of the quantitative designs, was adopted. The research conducted on fifth grade primary school students concluded that AR picture story books could reduce students' anxiety levels in mathematics learning. In the research conducted by Palanci (2023), which adopted a mixed research method, 7th grade students were taught for 5 weeks with AR-supported materials and it was observed that it had a positive effect on the students' mathematics anxiety, motivation, academic success and retention of knowledge. In line with the results obtained from the research, it was concluded that AR technology is an effective learning tool in mathematics education. Angraini et al. (2023) focused on evaluating basic mathematics skills that determine whether seventh grade students are ready to learn under teacher guidance. As a result of the research, it was seen that AR-supported teaching gave better results on students with low mathematics skill levels compared to traditional teaching. Koparan et al. (2023) focused on improving the spatial abilities of secondary school students using mobile devices and the sample consisted of fifth grade secondary school students. As a result of the research,

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it was seen that AR-supported teaching greatly improved students' learning outcomes. Although many of these studies have investigated the effect of AR-supported learning environments on students' academic achievement (Akın, 2022; Akkuş, 2021; Altıok, 2020, Rossano et al., 2020) and spatial abilities (Gün & Atasoy, 2017; Topraklıkoğlu, 2018; Yanuarto & Iqbal, 2022), few of these studies have investigated whether they have an impact on students' academic achievement as well as spatial abilities (del Cerro Velázquez & Morales Méndez, 2021). According to the results of the research conducted by Topraklikoğlu (2018) and del Cerro Velázquez and Morales Méndez (2021), it was seen that AR supported teaching contributed positively to the spatial ability progress of students. In the current study, AR applications were used in teaching the subject of geometric objects, which students had difficulty in concretizing, and in parallel with the studies, its effect on the academic success and development of their spatial abilities, which is the ability to think of objects in three dimensions in the mind was examined, and considering that spatial ability is a form of geometric thinking, it was examined whether it had an effect on students' van Hiele geometric thinking levels. In this regard, the research problems tried to be answered in the study are as follows;

• Does teaching the subject of geometric objects with augmented reality have an effect on the development of students' geometric thinking levels?

• Does teaching the subject of geometric objects with augmented reality have an effect on the development of students' spatial abilities?

• Does teaching the subject of geometric objects with augmented reality have an effect on students' academic success?

2. METHODOLOGY

2.1. Research Design

In this study, in which the impacts of the events designed with AR in teaching the subject of geometric objects on the progress of students' geometric thinking levels and spatial abilities were investigated, a weak experimental design was adopted from the experimental methods. Additionally due to the limitations of being a single branch in the school, a quantitative research approach was used to achieve the goals.

In the study, quantitative data were obtained with a single group weak experimental design. In the single-group pretest-posttest experimental design that does not include random assignment; the researcher investigates the effects of the independent variable on the dependent variable (Gay & Airasian, 2000). In the weak experimental design, the participants' measures of the dependent variable before the application were pre-test, and after the application, the same subjects were subjected to the same data collection tools as the post-test (Büyüköztürk et al., 2019).

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In this study, it was purposed to investigate whether the independent variable (AR applications) had an effect on the dependent variables (van Hiele geometric thinking levels, spatial abilities and academic achievements) and was carried out over a single group (Experimental Group).

The flow chart of the research process, which included 15 lesson hours, is given in Figure 1.

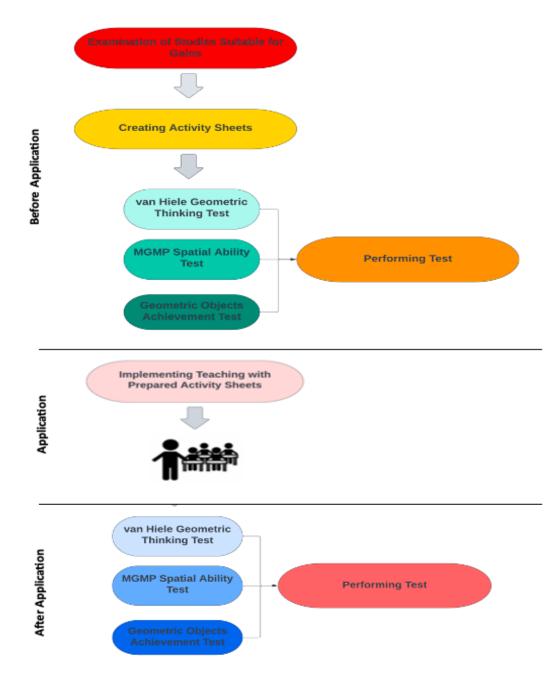


Figure 1. Flow Chart of the Research Process

2.2. Sample of Research

The study group of the research constitute of sixth grade students studying in a village secondary school affiliated to MEB in the Eastern Anatolia region in the 2021-2022 academic year. All activities were carried out in a sixth grade consisting of 8 girls and 7 boys, a sum of 15 students. Academic achievement of students is at a low level due to the lack of a suitable study environment and some disruptions in education and training.

In this study, convenient sampling method was preferred because the students in the secondary school institution where the researcher worked constituted the study group. Appropriate sampling is the election of the sample starting from the most reachable participants due to the boundedness in terms of time, money and workforce in the research (Büyüköztürk et al., 2012; Kılıç, 2013).

2.3. Data Collection Tools

2.3.1. van Hiele Geometric Thinking Test (vHGTT)

The Turkish version of the van Hiele Geometric Thinking Test was used in the study in which the effects of AR applications on the progress of students' geometric thinking levels were examined. Considering the students' levels, the first three levels were applied.

During her master's thesis, Duatepe (2000) adapted the 25-question multiple-choice vHGTT developed by Usiskin (1982) to designate the geometric thinking levels of students and carried out validity-reliability studies. Duatepe (2004) applied the first 15 items of the test to 102 seventh grade students during her doctoral thesis and calculated the Cronbach's alpha reliability coefficients as .82, .51, and .70 for the first three levels, respectively. In the data applied by İbili (2013) to sixth grade students, the Cronbach Alpha reliability coefficient for the first three levels was calculated between .60 and .77. In this study, vGHTT was applied to sixth grade students and the Cronbach Alpha reliability coefficient was calculated as .718 for the sum of the first three levels. Due to the hierarchical nature of vHGTT, it is necessary to pass the previous levels in order to be at any of the levels. However, a certain level can be determined, provided that the student has correctly answered at least 4 out of 5 questions (Usiskin, 1982).

According to Usiskin's (1982) scoring key for vHGTT:

Level 1: (1-5) 1 point if he/she solves (1-5) questions and meets the criteria

Level 2: 2 points if he/she solves (6-10) questions and meets the criteria

Level 3: If the (11-15) questions are solved and the criteria are met, 4 points are given.

According to the scoring key, a student can get a maximum of 1 point for the first level, 3 points for the second level and 7 points for the third level.

2.3.2. MGMP Spatial Ability Test

The MGMP Spatial Ability Test, which is the Turkish version of the MGMP Spatial Visualization Test, was used in the study in which the effects of AR applications on the development of students' spatial abilities were examined.

The MGMP Spatial Visualization Test was developed by Michigan State University mathematics faculty members to be used in the project called Middle Grades Mathematics Project. Consisting of 32 multiple-choice questions, this test was created with 5 choices and was planned to be applicable for the second level of primary education (Turğut, 2007).

In this test, which was translated into Turkish by Turğut (2007), after some items were removed from the test and new items were added with the help of expert opinions, a new test consisting of 29 questions with 4 options suitable for the level of secondary school students and a reliability coefficient of .830 was created by analyzing it in the ITEMANN program. Turğut (2007) named this new test the MGMP Spatial Ability Test (SAT). In this study, the Cronbach Alpha reliability coefficient for the MGMP Spatial Ability Test applied on sixth grade students was calculated as .739. The lowest 0 points and the highest 29 points can be obtained from the test.

2.3.3. Geometric Objects Achievement Test (GOAT)

The Geometric Objects Achievement Test was used in the study in which the effect of AR applications on the academic success of students in geometry was examined. This test, which was developed by Yılmaz (2019) to measure the permanence of what has been learned and the academic success of students, consists of 29 multiple-choice questions and is arranged with 5 choices. While the Geometric Objects Achievement Test (GOAT) was being developed, the sub-learning areas and achievements of the 6th grade geometric objects learning area were determined (Yılmaz, 2019).

The KR-20 value of the final achievement test was calculated as .928 and this value revealed that the reliability of the test was very high (Y1lmaz, 2019). In this study, the Cronbach Alpha reliability coefficient for the Geometric Objects Achievement Test (GOAT) applied to sixth grade students was calculated as .707. The lowest 0 points and the highest 24 points can be obtained from the test.

2.4. Period

The implementation time of AR activities was determined as 15 lesson hours, taking into account the time allocated to the subject of geometric objects in the mathematics curriculum. The research was conducted with the participation of 15 6th grade students. Throughout the application, the researcher provided the help and guidance the students needed to learn and checked whether their learning was achieved.

Before the application process started, students were divided into two or three study groups, taking into account the number of phones/tablets, and the AR application was installed on the phones/tablets used by the students.

Necessary support was provided to students who requested help regarding the use of .apk files on phones/tablets and the way the activities were implemented during the application. Although the instructions were given clearly in the activities on the activity sheets, verbal explanations were also made to the students about the content of the activity.

The activity sheets distributed at the end of the application were collected and the data of the research were collected through the applied tests. The same measurement tools were used on the same study group at the beginning and end of the application.

1 Lesson Hour

After uploading the .apk files of the relevant activities to the students' phones/tablets, the AR application was opened. The lesson started by reading the first question on the activity sheets. Students were made to think about the question prepared in accordance with the objective of "Understands that the number of unit cubes placed in a rectangular prism without any space is the volume of that object, and calculates the volume of the given object by counting the cubes given." and their answers to the question were received. Later, when we opened the relevant question on the AR application, the students were initially surprised to see the unit cubes filled into the rectangular prism-shaped box on paper in 3D form. This situation started to attract the attention of even the students who were not interested in the course and aroused curiosity in the students. The researcher went to each group one by one and received students' comments about the question and their thoughts about the solution. With the guidance, students were enabled to discover the concept of volume and to learn that the volume of a given object can be calculated with unit cubes by reasoning about the concept of volume.

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Figure 2. Imagine of the Application Process

2.5. Data Analysis

The analysis of quantitative data was organized and analyzed with the help of IBM SPSS Statistics version 26.0 data processing program. Before the application, data analysis was made for the mathematics course achievement levels of the study group and the Shapiro-Wilk test was applied to check the mathematics grades of the students in the study group were suitable for normal distribution. The reason for applying this test is that the number of students is less than 50.

As a result of the Shapiro-Wilk Test (Study group, p=.638; p>.05) analysis conducted for the 2021-2022 academic year first semester mathematics course grades of the students in the study group, it was seen that the data demonstrated a normal distribution. Afterwards, data collection tools were analyzed.

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In the analysis of the data, the total scores and averages of vHGTT, MGMP SAT and GOAT were calculated and examined. For the interpretation of the mean obtained from the vHGTT test, it is assumed that the vHGTT scale intervals are equal and the score range is (Maximum Value - Minimum Value / number of categories = (12-2)/5 = 2 (Sahin, 2013). If the mean score obtained from the vHGTT test is between 2-4, it is low level, if it is between 4.01-6 it is medium-low, if it is 6.01-8 it is moderate, if it is 8.01-10 it is medium-high, and between 10.01-12 it is high. For the interpretation of the mean obtained from the MGMP SAT test, it is assumed that the MGMP SAT scale intervals are equal and the score range is (Maximum Value – Minimum Value / number of categories = (25-5) / 5 = 4) 4 (Sahin, 2013). If he average score obtained from the MGMP SAT test is between 5-9, it was determined as low level, between 9.01-13 as medium-low level, between 13.01-17 as medium-level, between 17.01-21 as medium-high level, and between 21.01-25 as high level. For the interpretation of the mean obtained from the GOAT test, it is assumed that the GOAT scale intervals are equal and the score range (Maximum Value - Minimum Value / number of categories = (21-3) / 5 =3.6) is 3.6 (Sahin, 2013). If the average score obtained from the GOAT test is between 3-6.6, it was determined as low level, between 6.61-10.2 as medium-low level, between 10.21-13.8 as medium level, between 13.81-17.4 as medium-high level and between 17.41-21 as high level. With the data obtained from these three tests, the level of geometric thinking levels, spatial abilities and academic successes of the students in geometric objects was examined.

While analyzing the data, it was examined whether the sum scores of the participants from the tests showed a normal distribution in order to determine which of the parametric or non-parametric tests would be used. In order to decide the normality of the data, skewness and kurtosis values were examined and the Shapiro-Wilk test was applied. As a result of the analysis, it was seen that the skewness and kurtosis values were between -2 and +2 and the assumption of normality was provided (George & Mallery, 2010). As the skewness and kurtosis values supported the normal distribution, the Shapiro-Wilk test was used to check the convenience of the data to the normal distribution. The reason for applying this test is that the number of students is less than 50. The skewness and kurtosis coefficients of the students' total scores from VHGDT and MGMP UYT are given in Table 1.

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	Ν	Missing Values	Skewness Values	Kurtosis Values	Shapiro-Wilk Test Result
vHGTT (Pre Test)	15	0	.423	.399	.357
vHGTT (Post Test)	15	0	.607	634	.251
MGMP SAT (Pre Test)	15	0	.588	.900	.407
MGMP SAT (Post Test)	15	0	1.145	1.998	.174
GOAT (Pre Test)	15	0	.164	1.080	.423
GOAT (Post Test)	15	0	.732	1.065	.651

Table 1. Skewness and Kurtosis Values and Shapiro-Wilk Test results of vHGTT, MGMP SAT and GOAT Scores

When Table 1 was examined, it was seen that the data showed a normal distribution and it was decided that parametric tests could be used to compare the results of the research (Christensen et al., 2015; Seçer, 2017).

3. FINDINGS

Before the analysis of the tests applied in the study, it was checked whether the test results obtained showed a normal distribution. Since test results demonstrated normal distribution, dependent samples t-test was used from parametric tests to find answers to research problems. While the findings obtained from the applied tests were given, the data obtained before and after the application were also added and examined under three headings.

3.1. Findings Obtained for the van Hiele Levels of Geometric Thinking Test

The van Hiele Geometric Thinking Levels Test consists of 25 multiple-choice questions to measure the geometric thinking levels of students. However, since the levels of the test have a hierarchical structure and the sample of this study consists of secondary school students, only the first 15 questions of the test were used in line with the students' levels. The lowest score that can be received from this test is 0, and the highest score is 7. The descriptive statistics of the total number of correct answers on the vHGTT of the 15 students who participated in the study are given in Table 2.

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TEST	n	Number of Questions	Minimum	Maximum		SS
vHGTT (Pretest)	15	15	2	10	5.075	1.292
vHGTT(Posttest)	15	15	3	12	7.351	2.933

Table 2. Descriptive Statistics of vHGTT Average of Total Number of Correct Answers

If the mean score obtained from vHGTT is between 2-4, it was determined as low level, between 4.01-6, as medium-low level, between 6.01-8, as medium level, between 8.01-10 as medium-high, and between 10.01-12 as high level. As can be seen in Table 8, the average score of the students in the pretest of the vHGTT was determined as 5.075. Since 4.01<5.075<6, the mean score of the pretest of vHGTT is moderate-low. The fact that the mean score of the pretest of the vHGTT was at a medium-low level indicates that the van Hiele geometric thinking levels of the 6th grade students participating in the research were between low and moderate before the application.

As seen in figure 1, the mean score of the students in the posttest of the vHGTT was determined as 7.351. Since 6.01<7.351<8, the mean score of vHGTT is moderate. The fact that the mean score of the posttest of the vHGTT was at a moderate level indicates that the van Hiele geometric thinking levels of the 6th grade students participating in the study were at a moderate level after the application.

The frequency distributions of the scores of the students' van Hiele geometric thinking levels in the pretest are given in Figure 3.

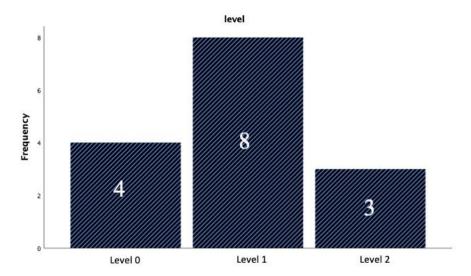


Figure 3. Frequency Distribution of Student Numbers According to vHGTT Pretest Results

The frequency distributions of the scores of the students' van Hiele geometric thinking levels in the posttest are given in Figure 4.

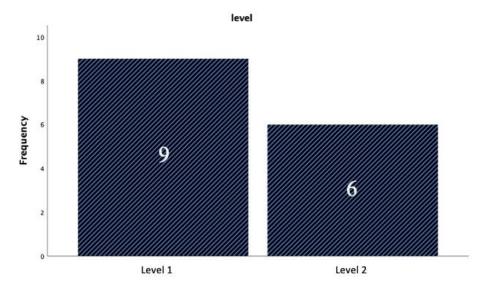


Figure 4. Frequency Distribution of Student Numbers According to vHGTT Posttest Results

When the frequency distributions in Figure 3 and Figure 4 are examined, it is seen that while there were 4 students at Level 0 in the pretest, there were no students at Level 0 in the posttest. In other words, all of the students at Level 0 reached a higher level after the application. While there are 8 students at Level 1 in the pretest, there are 9 students in the posttest. While there are 3 students at Level 2 in the pretest, there are 6 students at Level 2 in the posttest. This means that 4 students at Level 0 could be promoted to Level 1, and some of the existing students at Level 1 could be promoted to Level 2, which is a higher level. Due to the characteristics of the 3rd Level, namely the Formal Inference Level, the student's knowledge and skill level reaches the level of proving at this level. Students in this period begin to grasp the importance of inference, understand the role of axiom theorem and proof. Based on this, it was observed that there were no students in the pretest at the 3rd level. Considering the characteristics of the 3rd level, it can be said that this situation is normal (Fidan & Türnüklü, 2010). In addition, it was occurred that the level of the students incremented in the posttest compared to the pretest.

As a result of the analysis of skewness and kurtosis values, and Shapiro-Wilk test (Pretest, p =.357; p>.05, Posttest p = .251; p>.05) according to vHGTT pretest-posttest scores, the data showed normal distribution. It has been determined that the occurred data will be analyzed

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with parametric tests (Christensen et al., 2015; Seçer, 2017). For this reason, paired sample ttest was used in pairwise comparison for vHGTT and the results are given in Table 3.

Variable	n		SS	t	sd	р
Pretest Score	15	1.133	1.060	-2.32	14	.036
Posttest Score	15	1.800	1.014			

Table 3. Paired Sample T Test Result of vHGTT Results

When Table 3 is examined, according to the paired sample t-test results, it is seen that there is a statistically significant difference between the students' pretest success scores and posttest success scores in favor of the posttest and the effect size of the difference is medium. $(t_{(14)} = -2.32, p<.05; Cohen's d=.643).$

3.2. Findings Obtained on the MGMP Spatial Ability Test

The MGMP Spatial Ability Test consists of 29 multiple-choice questions to measure students' spatial abilities. The lowest score that can be received from this test is 0, and the highest score is 29. The descriptive statistics of the MGMP SAT scores of the 15 students who participated in the study are given in Table 4.

TEST	N	Number of Questions	Minimum	Maximum		SS
SAT (Pretest)	15	29	5	21	12.333	4.029
SAT (Posttest)	15	29	7	25	13.600	4.436

 Table 4. Descriptive Statistics of MGMP SAT

Average score from MGMP SAT; It was determined as low level between 5-9, medium-low level between 9.01-13, medium level between 13.01-17, medium-high level between 17.01-21 and high level between 21.01-25.

As seen in Table 4, the average score of the students regarding the pretest of the MGMP SAT was determined as 12,333. Since 9.01<12.333<13, the mean score of the pre-test of the MGMP SAT is medium-low. The fact that the mean score of the pre-test of the MGMP SAT is at a medium-low level indicates that the spatial abilities of the 6th grade students participating in the research are between low and moderate before the application.

As can be occured in Table 4, the average score of the students in the post-test of the MGMP SAT was determined as 13.600. Since 13.01<13.600<17, the mean score of MGMP SAT is

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moderate. The average score of the posttest of the MGMP SAT shows that the spatial abilities of the 6th grade students participating in the research are at a moderate level after the application.

Normal distribution of the data as a result of the analysis of skewness coefficients, kurtosis coefficients and Shapiro-Wilk test (Pretest, p = .407; p>.05, Posttest p = .174; p>.05) according to MGMP SAT pretest – posttest scores It has been determined that the data obtained will be analyzed with parametric tests. (Christensen et al., 2015; Seçer, 2017). For this reason, Paired Sample t-test was used in pairwise comparison for MGMP SAT and the results are given in Table 5.

Variable	n		SS	t	sd	р
Pretest Score	15	12.333	4.029	-2.244	14	.042
Posttest Score	15	13.600	4.436			

Table 5. Paired Sample T Test Result of MGMP SAT Results

When Table 5 is examined, according to the paired sample t-test results, it is seen that there is a statistically significant difference between the students' pretest success scores and posttest success scores in favor of the posttest and the effect size of the difference is small. ($t_{(14)} = -2.244$, p<.05; Cohen's d=.299). While the pretest mean score of the students is 12.33, the posttest mean score is 13.60. It can be said that mathematics teaching with AR applications contributes to the development of students' spatial abilities due to the increase in their achievement score averages.

3.3. Findings Obtained for the Geometric Objects Achievement Test

The Geometric Objects Achievement Test constitute of 24 multiple-choice questions to measure the academic success of students. The lowest score that can be received from this test is 0, and the highest score is 24. The descriptive statistics of the GOAT scores of the 15 students who participated in the study are given in Table 6.

TEST	n	Number of Questions	Minimum	Maximum		SS
GOAT (Pretest)	15	24	3	14	7.533	2.948
GOAT (Posttest)	15	24	5	21	11.666	3.940

Table 6. Descriptive Statistics of GOAT

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Average score from GOAT; It was determined as low level between 3-6.6, medium-low level between 6.61-7.2, medium level between 7.21-10.8, medium-high level between 10.81-14.4 and high level between 14.41-21.

As can be occurred in Table 6, the average score of the students in the pretest of the GOAT was determined as 7.533. Since 7.21<7.533<10.8, the mean score for the pretest of GOAT is moderate. The average score of the GOAT pretest indicates that the academic success of the 6th grade students participating in the research on geometric objects is at a moderate level before the application.

As occurred in Table 6, the average score of the students in the posttest of GOAT was determined as 11,666. Since 10.81<11.666<14.4, the mean score for GOAT is medium-high. The fact that the sum score of the posttest of GOAT is at a medium-high level indicates that the academic success of the 6th grade students participating in the study in geometric objects is between moderate and high after the application.

As a result of the analysis of skewness coefficients, kurtosis coefficients and Shapiro-Wilk test (Pretest, p = .423; p>.05, Posttest p = .651; p>.05) according to GOAT pretest-posttest scores, the data showed normal distribution. It has been determined that the obtained data will be analyzed with parametric tests (Christensen et al., 2015; Seçer, 2017). For this reason, paired sample t-test was used in pairwise comparison for GOAT and the results are given in Table 7.

Variable	n		SS	t	sd	р
Pretest Score	15	7.533	2.948	-3.952	14	.001
Posttest Score	15	11.666	3.940			

Table 7. Paired Sample T Test Result for GOAT Results

When Table 7 is examined, according to the paired sample t-test results, it is seen that there is a statistically significant difference between the students' pre-test success scores and post-test success scores in favor of the post-test and the effect size of the difference is large.. ($t_{(14)} = -3.952$: p<.05; Cohen's d=1.187). While the pre-test mean score of the students is 7.533, the post-test mean score is 11,666. It can be said that mathematics teaching with AR applications contributes to the improvement of students' academic success due to the increase in their achievement score averages.

4. RESULTS AND DISCUSSION

In this section, the results received from the research are presented under three headings and these results are discussed by comparing them with the studies in the literature.

4.1. Results on the Effect of Using AR Applications in Geometry Teaching on the Development of Students' Geometric Thinking Levels

The first sub-problem of the research is "Does teaching the subject of geometric objects with AR have an effect on the development of students' geometric thinking levels?" As a result of the analyzes for the first sub-problem, a positive difference was found between the AR applications and the vHGTT pretest-posttest scores of the teaching students ($t_{(14)} = -2.32$, p<.05). This finding shows that AR supported instruction is effective on students' geometric thinking levels and significantly increases students' geometric thinking levels. The reason for this circumstance can be demonstrated as the fact that AR applications attract the attention of students, arouse curiosity in students and provide active participation in the lesson, while the concretizing aspect of AR applications can also contribute to the reasoning that develops with the concretization of abstract concepts in the student's mind. When the literature is investigated, it is occurred that the studies examining the impacts of geometry teaching with AR applications on students' geometric thinking levels are limited. İbili (2013) examined the effect of the educational environment promoted by AR on students' achievement and attitudes and developed a software with the help of AR to observe the static drawings in geometric objects more dynamically. Unlike this study, in the study that had experimental and control groups, van Hiele geometric thinking levels of students were examined and although there was no significant difference between the geometric thinking levels of the control group students according to the pretest-posttest data, a positive significant difference was found between the geometric thinking levels of the experimental group students as in this study. As a result of the research, it was concluded that AR-supported instruction positively influence students' geometric thinking levels. The reason for this can be shown as the fact that AR applications incremented the effectiveness of learning environments with their concretizing aspect and activate students' geometric thinking and reasoning. Widyasari and Mastura (2019) also aimed to improve students' geometric thinking skills with the help of AR, and in parallel with İbili (2013), they investigated students' geometric thinking levels in a study with experimental and control groups. As in this study, it has been concluded that the learning

environment in which AR applications are used has an effect on students' geometric thinking levels.

4.2. Results on the Effect of Using AR Applications in Geometry Teaching on the Development of Students' Spatial Abilities

The second sub-problem of the research is "Does teaching the subject of geometric objects with AR have an effect on the development of students' spatial abilities?". As a result of the analyzes for the second sub-problem, a significant difference was found between the AR applications and the MGMP SAT pretest and posttest scores of the teaching students ($t_{(14)} =$ -2.244, p<.05). This finding shows that AR-supported instruction is effective on students' spatial abilities and significantly increases students' spatial abilities. When the literature is examined, results similar to the results of this study have been obtained in several studies examining the impacts of geometry teaching with AR applications on students' spatial abilities. In the study conducted by Gecü-Parmaksız (2017), the impact of geometry teaching supported by AR applications on the progress of students' spatial skills was examined. In this study, which has experimental and control groups, unlike our research, the spatial abilities of the students were determined according to the Pictorial Rotation Test and the Spatial Perception Test, and as a result of the research, it was concluded that teaching with AR applications positively affected the spatial skills of the students. As in this research, in the study conducted by Topraklıkoğlu (2018), who only had an experimental group, AR activities were designed and the effect of geometry teaching supported by these AR activities on the development of students' spatial abilities was examined. The spatial abilities of the students were determined according to the MGMP SAT and as a result of the research, it was concluded that the teaching with AR activities positively affected the spatial abilities of the students in parallel with the research we conducted. Yanuarto and Iqbal (2022) examined the effect of AR-enhanced learning environments on mathematical spatial ability, and as a result of the research, they concluded that AR-supported instruction contributed positively to students' mathematical spatial abilities, as in this study. In the study conducted by Koparan (2023), it was aimed to design, develop and reveal the effectiveness of an AR material to improve the mathematical spatial abilities of secondary school students. As in this research, the effect of spatial ability on mathematics learning was mentioned and AR materials were designed accordingly. As a result of the research, it was concluded that integrating AR

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materials with spatial skills into the teaching-learning process revealed a significant difference on students' academic achievements and attitudes.

4.3. Results on the Effect of Using AR Applications in Geometry Teaching on Students' Achievement

The third sub-problem of the research is "Does teaching the subject of geometric objects with AR have an effect on the academic achievement of the students?" As a result of the analysis for the third sub-problem, a statistically significant difference was found between the AR applications and the GOAT pretest-posttest scores of the teaching students in favor of the posttest ($t_{(14)} = -3.952$, p<.05). This finding shows that as a result of the use of AR applications in the teaching of the sixth grade geometric objects subject, AR supported teaching has an effect on the academic achievement of the students and significantly increments the academic achievement of the students. As a matter of fact, although students have difficulties in mathematics subjects consisting of abstract concepts, AR applications attract the attention of students thanks to their visualizing and concretizing quality, and thus, interest and active participation in the lesson are realized, and therefore it can be said that this brings academic success. Accompanied by the findings obtained from the research, it shows that teaching environments supported by AR applications have effective results in learningteaching processes. Similar results were obtained in several studies in the literature examining the impact of AR applications on student success. Liu et al. (2018) investigated the impact of geometry teaching supported by AR-based learning application on student attitude and success. As a result of the research, it was stated that AR applications had a positive impact on student attitudes and achievements. Altiok (2020) investigated the impacts of mobile AR-assisted teaching environments on students' views and success on the AR process. As a result of the research, it was stated that the academic achievements of AR applications increased significantly and had a positive impact on their opinions. Cai et al. (2020) stated in the study that the impact of mobile AR-based learning environments on students' learning was investigated, that students had positive attitudes towards AR applications, and that they found AR applications fascinating, interesting, motivating and more productive. He also presented that AR applications had a statistically positive impact on students' learning. del Cerro Velázquez and Morales Méndez (2021)) investigated the impact of GeoGebra AR supported learning environments on students' spatial ability and academic achievement. As a result of the research, it is suggested that teaching and learning processes

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supported by AR applications for teaching mathematics subjects improve students' spatial abilities and academic performance.

Considering the literature, it is pointed out that AR applications can be used as an effective material in learning-teaching processes (Fleck & Simon, 2013; Medicherla et al., 2010; Schutera et al., 2021; Tian et al., 2014; Topraklıkoğlu, 2018; Yanuarto & Iqbal, 2022), and studies have been found that positively affect students' academic achievement (Estapa ve Nadolny; 2015; Fidan, 2018; Chen, 2019; Liu et al., 2018; Rossana et al., 2020). Within the framework of these studies, it can be stated that AR supported learning environments increase success. On the other hand, it has been stated that AR-supported learning environments do not always increase success, but they have a facilitating effect on learning with their concretizing aspect (Akın, 2022; Martin et al., 2009). In AR-supported learning environments, it can be shown that AR technology arouses students' curiosity and that it brings the continuity of class participation in increasing the academic success of students (Akın, 2022). It is important to see that it creates an opportunity for the appearance of 3D objects on the real world image has a fascinating effect on students in accordance with the structure of AR applications, (Rossano et al., 2020), helping students to embody abstract subjects (İbili, 2013), providing active participation in the lessons (Hanson & Shelton, 2008), making students follow the lesson by arousing excitement and curiosity (Mahadzir & Phung, 2013) and provides an opportunity to offer fun and participatory learning experiences where the real world and the virtual are combined (Topraklıkoğlu, 2018).

5. RECOMMENDATIONS

In this section, the findings received as a result of the research and the recommendations made in the light of the results are given.

It has problems with internal validity due to poor experimental design, lack of other controlled procedures, and imprecise arguments (Sözbilir, 2012). However, due to the fact that there is only one branch in the school where the researcher works, a weak experimental design had to be preferred in this study. Therefore, working with a single group for geometry teaching supported by AR applications, as in this study, is a limitation. In a future research, the variables examined in this research can be examined in a quasi-experimental design by creating an experimental and control group.

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This research was conducted for 6th grade students and as a result of the research, it was seen that teaching with AR applications had a positive impact on students' geometric thinking levels, spatial abilities and academic achievements. It is a matter of curiosity whether it will affect students in different age groups in the same way. Therefore, in other studies, assisted education with AR applications can be done at different education levels (primary education, secondary education, higher education) and its effects on different variables can be investigated.

In the research, the effect of the use of AR applications on the teaching of the 6th grade geometric objects subject was examined and it was seen that it contributed to their academic achievement. One of the reasons for this is that the learning environments created with AR applications attract the attention of students. In a future research, the effect of teaching different subjects or different courses in learning environments created with AR applications on students' academic success and spatial abilities can be investigated.

In this research, Unity game engine and Vuforia plugin were used for AR application. The researcher was trained to learn and apply these software. Since these softwares were designed for game development, the researcher had difficulty in the initial stage of education. Whereas GeoGebra dynamic mathematics software is known by many mathematics teachers. In a future research, AR applications can be developed using other AR software such as GeoGebra AR.

In this research, it is seen that the learning environment created with AR applications has a positive effect on the academic success of students. It is a matter of curiosity whether this effect has a relationship with students' attitudes and motivations towards the lesson. Therefore, in a research to be conducted, it can be examined whether the AR supported teaching environment has an impact on students' attitudes and motivations towards the course and AR technology.

In AR applications developed for research, 3D objects are designed in a way that does not allow students to make changes on them (such as zooming in, zooming out, moving and rotating by touch). For the objects created in AR applications to be designed in future research, a study can be conducted that includes multi-touch gestures and the impact of this application on the academic success of students can be examined.

6. ABOUT THE AUTHORS

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